Optimal Bank Liability Structure

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Abstract

We develop a model of capital/liability structure of value-maximizing banks that optimally respond to changes of regulatory environment. Since it nests the standard model of capital structure for non-financial firms as a special case, our model offers insights into the distinctive properties of banks. By analytically solving the optimal liability structures of unregulated banks and FDIC-insured banks as well as banks that are subject to regulatory equity requirement, we produce the following results.

In the absence of regulation, banks take high leverage, both in the form of deposits and subordinated debt. The special driving forces of high leverage are the banks’ low asset volatility and the income from serving deposit accounts. The subordinated debt is found to be important in banks’ liability structure—holding zero subordinated debt is never optimal for a bank. However, a bank that optimizes its liability structure should not have too much subordinated debt; the optimal level is to set the endogenous default of debt to coincide with the point at which depositors choose to run. In this optimal choice of liability structure, subordinated debt does not protect deposits from bankruptcy.

The introduction of FDIC raises the market value of banks, even when banks are charged a fair insurance premium. With deposit insurance, a bank issues more deposits but reduces subordinated debt to ensure that endogenous default still coincides with bank closure. In the optimal liability structure, subordinated debt does not protect FDIC from losses in covering deposit insurance obligation when the bank is closed. After cutting back subordinated debt in response to the introduction of FDIC, the deposit expansion still results in a higher leverage. This optimal response dampens the reduction of expected bankruptcy loss that could be potentially brought by the FDIC insurance program.

Although the tax benefit of leverage is not a reason for banks to be different from other companies, corporate tax is more important for banks liability structure because banks take much higher leverage than non-financial firms. Our model offers a coherent framework to link bank optimal leverage to corporate tax rate. Based on our model, the optimal leverage of banks is lower in an economy with lower corporate tax rate. Moreover, the effects of corporate tax rate on bank leverage are almost the same for unregulated banks and FDIC-insured banks. The link lays a stepping stone for further welfare analysis of the benefit and cost of tax policy reforms.

The optimal response of banks to regulatory capital requirement is more complicated. Obviously, there should be no response if a bank’s optimal liability structure automatically satisfies the requirement. For a set of parameters for a typical bank, the optimal structure meets the 4% equity requirement of Basel II, but not the 7% equity requirement of Basel III. In order to meet a regulatory equity requirement of 7 percent or higher, the typical bank must trim both deposits and subordinated debt. However, the optimal reduction of leverage in response to equity requirement still ensures that subordinated debt does not protect deposits.
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1 Introduction

The banking industry has experienced several major crises in the past, and many of these crises have focused regulators’ attention on the inadequacy of equity capital held by the banks, which might have exacerbated the risk of bank runs or bank failure. Consequently, regulations were often introduced and revised after the banking crises. After the frequent bank runs experienced during the Great Depression, the Banking Act of 1933 was signed into law to create the Federal Deposit Insurance Corporation (FDIC). After the world-wide financial crisis and Great Recession of 2007–2009, the banking industry faced a sweeping change in its regulatory environment. In the U.S., the Dodd-Frank Wall Street Reform and Consumer Protection Act (often referred to as the Dodd-Frank Act) was signed into law in 2010 to bring regulatory reforms in a wide range of areas from FDIC deposit insurance to stress tests of banks’ capital adequacy. Internationally, regulators agreed in 2011 on more stringent capital adequacy standards for banks, which are collectively named as Basel III. It lifts the tier-1 equity capital requirement for all banks from 4 percent in Basel II to 7 percent and to even higher levels for banks that are designated as systemically important.

Since the introduction of Dodd-Frank Act and Basel III, regulators around the world have gradually rolled out changes to the regulation of banks’ capital structure, and the shape of bank regulation is still evolving. The Swiss National Bank has raised the capital requirement to 19 percent for its two largest banks. The European Union has set up its additional capital requirement in accordance with Basel III. The Federal Reserve, the FDIC, and the Office of Comptroller of the Currency (OCC) have issued the interim rule on raising capital requirement for the U.S. banks and foreign banks operating in the U.S. The Volcker Rule drafted by the U.S. regulators aims to control the risk of bank assets more effectively. Proposals for further regulatory changes are abundant in the academic literature. Some have argued for raising equity requirement to a level as high as 20 percent.\(^1\) There are also antithetical views on whether banks should hold subordinated debt.\(^2\) A drastic proposition is to reduce corporate tax for banks.\(^3\)

The recent changes of regulation and the proposals for additional changes call for careful analyses of the effects of regulatory changes. Each regulatory change tends to fix a particular observed broken factor in the bank’s assets or liability. For example, the deposit insurance

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\(^1\)See the book by Admati and Hellwig (2013) and a non-technical article by Admati, DeMarzo, Hellwig, and Pfleiderer (2011).

\(^2\)For example, Bulow and Klemperer (2013) suggest that banks should hold no subordinated debt. In their view, the only securities issued by banks should be equity or securities convertible to equity. In contrast, the Fed governor Daniel Tarullo (2013) argued that a requirement of holding long-term subordinated debt would improve capital structure and resolution of banks.

\(^3\)Citing academic studies of corporate tax as incentives of taking debt, Fleischer (2013) suggests that reducing corporate tax for banks will make banks safer. Schahndlbauer (2013) finds evidence that an increase in tax rate causes the average bank to increase their non-depository debt by about 5.9\% one period prior to the enactment of the tax change.
intends to address bank runs caused by the fear of losing deposits, which are the major source of financing for banks. The equity requirement intends to address the high leverage of banks. The Volcker Rule addresses the risks of the assets held by the banks and intends to cut down the risk of large losses in assets that bank equity must absorb. However, these intentions implicitly assume that banks will keep other things the same, ignoring the overall optimal response of banks that can change its assets and liabilities. With deposit insurance, it can be optimal for a bank to finance assets with more deposits, exposing the bank to a higher risk of being closed if it incurs large losses. With a higher equity requirement, a bank may find it more optimal to reduce banking services. With lower asset risk, banks may be better off by increasing leverage. More broadly, it is unclear whether the optimal response of the bank will undo or significantly diminish the intended effects of a regulatory change. It is even possible that the regulatory changes may result in unintended consequences.

In order to understand banks’ optimal responses to regulatory changes, we develop a parsimonious model of optimal capital/liability structure of banks. Following the traditional literature in corporate finance, we assume a bank maximizes its total market value by financing its cash-flow generating assets through issuing debt and equity. Our assumption is consistent with the principle that the bank management should act in the interest of its claim-holders. We do not assume that banks choose their liability structure to maximize a measure of social welfare such as reducing systemic risk and increasing banking services. Analysis of social welfare associated with alternative bank regulations is unquestionably important, but understanding the optimal response of banks to regulatory changes is a necessary first step for a proper social welfare analysis of bank regulation. This is the primary goal of our paper.

In our model, banks take deposits, issue subordinated debt, and are owned by equity holders. Banks’ assets are risky and they generate cash flows. A distinctive characteristic of banks is that banks take deposits and provide account services to their depositors. Bank deposit is different from other forms of debt partly because banks earn income from the provision of account services to depositors. More striking differences between deposits and other debt are that depositors can run, deposits can be FDIC insured, and deposit-taking banks are subject to special regulations. Our model incorporates these features of banks explicitly. Most importantly, bank run by rational depositors and bank closure by rule-following regulators are fully reflected in equity holders’ endogenous choice of defaulting on debt in order to maximize bank’s market value. Deposit insurance premiums are endogenously determined by FDIC based on the risk in the bank’s asset and liability structure. Extending the framework pioneered by Merton (1974, 1977) and Leland (1994), we analytically solve for the optimal bank liability structure under various regulatory environments, which allows us examine the optimal responses of banks to the changes in regulation. Our model produces the following results.

In the absence of regulation, banks take high leverage, both in the form of deposits and subordinated debt. The special driving forces of high leverage are the banks’ low asset
volatility and the income from serving deposit accounts. The subordinated debt is found to be important in banks’ liability structure—holding zero subordinated debt is never optimal for a bank. However, a bank that optimizes its liability structure should not have too much subordinated debt; the optimal level is to set the endogenous default of debt to coincide with the point at which depositors choose to run. In this optimal choice of liability structure, subordinated debt does not protect deposits from bankruptcy.

The introduction of FDIC raises the market value of banks, even when banks are charged a fair insurance premium. With deposit insurance, a bank issues more deposits but reduces subordinated debt to ensure that endogenous default still coincides with bank closure. In the optimal liability structure, subordinated debt does not protect FDIC from losses in covering deposit insurance obligation when the bank is closed. After cutting back subordinated debt in response to the introduction of FDIC, the deposit expansion still results in a higher leverage. This optimal response dampens the reduction of expected bankruptcy loss that could be potentially brought by the FDIC insurance program.

Although the tax benefit of leverage is not a reason for banks to be different from other companies, corporate tax is more important for banks liability structure because banks take much higher leverage than non-financial firms. Our model offers a coherent framework to link bank optimal leverage to corporate tax rate. Based on our model, the optimal leverage of banks is lower in an economy with lower corporate tax rate. Moreover, the effects of corporate tax rate on bank leverage are almost the same for unregulated banks and FDIC-insured banks. The link lays a stepping stone for further welfare analysis of the benefit and cost of tax policy reforms.

The optimal response of banks to regulatory capital requirement is more complicated. Obviously, there should be no response if a bank’s optimal liability structure automatically satisfies the requirement. For a set of parameters for a typical bank, the optimal structure meets the 4% equity requirement of Basel II, but not the 7% equity requirement of Basel III. In order to meet a regulatory equity requirement of 7 percent or higher, the typical bank must trim both deposits and subordinated debt. However, the optimal reduction of leverage in response to equity requirement still ensures that subordinated debt does not protect deposits.

The road-map for this paper is as follows. In section 2, we develop the model of bank liability structure in the context of several regulatory environments. In section 3, we characterize bank optimal liability structures in those regulatory environments. In section 4, we examine the effects of regulatory changes when banks are assumed to adjust liability structure optimally. Section 5 concludes the paper by placing our work in the context of the literature on bank capital.
2 Liability Structure and Regulation

Banks share some common characteristics with non-financial firms: both have access to cash flows generated by their assets and both finance their assets by issuing debt and equity. Banks, however, differ from non-financial firms in that they take deposits and provide liquidity services to their depositors through check writing, ATMs, and other transaction services such as wire transfers, bill payments, etc. The banking business of taking deposits and serving accounts is heavily regulated in most countries. In the U.S., a large part of deposit accounts is insured by the Federal Deposit Insurance Corporation (FDIC), which imposes additional regulations on banks. Deposits and the associated account services, FDIC insurance, and bank regulation distinguish banking business from other non-financial corporate business and set the bank capital decision apart from that of other firms.

A typical firm operates in a market with two important frictions: corporate taxes and costs associated with default/bankruptcy. These two frictions are the most crucial for a general firm in its choice of capital and liability structure, as recognized in the literature originating from Modigliani and Miller (1963) and Baxter (1967) and analyzed more recently in Leland (1994). While banks face these same frictions, they have to also simultaneously incorporate other considerations such as the potential of a run by depositors, FDIC deposit insurance premium, charter authority’s closure rules, and bank regulations on capital requirements in determining their optimal capital and liability structure. Figure 1 illustrates assets and liabilities of a typical bank. We will analyze each part of bank liability structure thoroughly after discussing our model of bank assets.

2.1 Bank Assets and Liabilities

In our model, the bank owns a portfolio of risky assets that generate cash flows. The portfolio of assets is valued at $V$, which is the major part on the asset side of Figure 1. The asset is risky, and its value follows a stochastic process. The instantaneous cash flow of the asset is $\delta V$, where $\delta$ is referred to as the rate of the cash flow. The risk of the asset is represented by the volatility of the asset value and denoted by $\sigma$. We assume that the portfolio of assets is given exogenously. Although this assumption rules out interesting issues of endogenous asset substitution, we will later examine the incentives for banks to alter asset riskiness or cash flow. Following Merton (1974) and Leland (1994), we assume that all investors have the full information about the asset portfolio value.

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4Following Merton (1974) and Leland (1994), we assume the stochastic process is a geometric Brownian motion, which is described by equation (19) in Appendix.

5The literature has pointed out that debt may create incentives to substitute assets with higher risk (e.g., Green, 1984, and Harris and Raviv, 1991) and FDIC insurance may also make for such incentive (e.g., Pennacchi, 2006, and Schneider and Tornell, 2004).

6In reality only accounting values of assets are reported quarterly, and thus the assumption of full information sets aside the disparity between accounting value and intrinsic value. Under this assumption, we
The most distinctive feature of the liability structure of a bank is its function to take deposits from households or businesses and pay interest on such deposits. Deposits, which are shown as the first part on the liability side in Figure 1, are often the single most important source of funds to finance the bank’s assets. We denote by $D$ the total amount of deposits that the bank takes. There are two ways to make a deposit a safe debt: (1) depositors withdraw their money early enough to ensure that the bank has enough assets to redeem depositors, or (2) the bank purchase insurance that guarantees the depositors in full. The first way induces bankruptcy cost due to a bank run, and the second way requires that the bank pay insurance premium, denoted by $I$. We will later discuss bank run and deposit insurance in more detail.

If deposits can be withdrawn at anytime without risk, the fair interest rate on deposits should be the risk-free rate. On deposit accounts, however, banks typically pay lower interest rate. Depositors accept lower interest rate because they receive the service of maintaining accounts and transacting payments, such as writing checks and withdrawing cash at ATM. Bank also charge fees for services such as money transfers, overdrafts, etc. Suppose the risk-free interest rate is $r$. If the net income (from the bank’s perspective) for serving deposit accounts is $\eta$ per dollar of deposits, the net interests on deposits is $C = (r - \eta)D$. The parameter $\eta$ plays a crucial role in our model. It represents a sacrifice to the required rate of return that the households are willing to accept. The sacrifice distinguishes deposits from other form of debt. If deposits are risk-free because of deposit insurance or due to the ability of depositors to run before the bank fails, the bank’s total cash outflow on deposits is $I + C$.

An important form of debt, in addition to deposits, issued by banks is the subordinated debt. It is the second part of the liability side in Figure 1. Subordinated debt pays coupon until bankruptcy, at which it has a lower priority than deposits in its claims on the liquidation value of a bank. The lower priority potentially protects deposits at bankruptcy. For that reason, long-term subordinated debt is viewed as tier-2 capital of the bank by regulators. A perpetual subordinated debt like “noncumulative preferred equity” is labeled as “additional tier-1 capital” in Basel III. This, however, comes at a price: the long-term debt yields will capture a credit spread above the risk-free rate to compensate them for bearing the risk of bankruptcy. This credit spread arises endogenously in our model, depending on the asset risk and the (endogenous) leverage of the bank. Thus, a bank’s choice of liability structure affects the credit spread. Let $s$ be the credit spread, which we will solve in our model endogenously, and $D_1$ be the face value of subordinated debt. The coupon on subordinated debt is $C_1 = (r + s)D_1$, which lasts until bankruptcy. We will discuss bankruptcy later in more detail.

The bank is owned by its common equity holders, who garner all the residual value and may interpret $V$ as the fair accounting value. Furthermore, if the assets have relatively homogenous risk, we may also interpret $V$ as the value of risk-weighted assets.
earnings of the bank after paying the contractual obligations on deposits and subordinated debt. The first slice of value that equity owners lay claim to is the difference between assets and debts: $V - (D + D_1)$, which is also on the liability side in Figure 1. This is sometimes referred to as tangible equity or book-value of equity, which is the value the equity holders would receive if bank assets are liquidated at fair value and all debts are paid off at par. A larger book-value of equity means deposits and subordinated debt are less likely to suffer a loss. Hence, regulators regard it as bank capital of the highest quality and refer to it as core tier-1 capital.

Equity holders are also rewarded by all future earnings of the bank. The present value of the future earnings is the bank’s charter value, which is the bottom part on asset side in Figure 1. Part of the earnings comes from the service income on deposit accounts: $\eta D$, but the earnings associated with deposit account should exclude insurance premium $I$ if deposits are insured. Another part of the earnings is the saving from corporate tax. Let $\tau$ be corporate tax rate. Since costs of debt financing are deductible from earnings for tax purposes, the total tax saving is $\tau(I + C + C_1)$. Therefore, the dividend paid to equity holders is the difference between asset cash flow and the net total outflow of cash associated with deposits and subordinated debt: $\delta V - (1 - \tau)(I + C + C_1)$. Since equity value depends on its dividend, it is affected by the liability structure. In banks with deposit insurance, the liability structure is characterized by the triplet $(I, C, C_1)$, whereas in banks without deposit insurance, the pair $(C, C_1)$ typifies the liability structure.

### 2.2 Bank Run, FDIC, and Equity Requirement

A natural consequence of borrowing using deposits is the risk that the depositors may run. This is an important way in which banks differ from most non-financial firms. As experienced in the crises of U.S. banking history and theorized by Diamond and Dybvig (1983), depositors may “run” if they believe that the banks may have difficulty in repaying their deposits in a prompt and timely fashion upon their demand.\(^7\) When depositors run, the bank will be closed and its assets have to be liquidated. Assume that the market liquidation costs through bankruptcy courts, which includes dead-weight losses associated with the liquidation process and legal expenses, is a fraction, $\alpha$, of the asset value $V_a$ at bankruptcy, leading to a liquidation value of $(1 - \alpha)V_a$. While deposits bring the benefit of account service income $\eta$, the cost associated with deposits is bankruptcy. With full information, it is rational for the depositors to run just before the bank decides to liquidate. The depositors will never wait if they believe that the value after liquidation is below the total deposits $D$. In addition, it may be reasonable to assume that the depositors, worrying about the delays that may ensue when the bank files for bankruptcy, might actually wish to run even earlier.

\(^7\)In September 2007, Norther Rock, a U.K. Bank, experienced a run on its deposits, and had to be nationalized in 2008. See Shin (2008) for a cogent analysis of Northern Rock run.
We assume that depositors will run when asset value drops to $V_a = D / (1 - \alpha)$. Since a bank is closed when depositors run, by letting $\kappa = 1 / (1 - \alpha)$ we can interpret $\kappa D$ as the threshold for bank to close due to bank run.

The establishment of the Federal Deposit Insurance Corporation (FDIC) is widely regarded as a deterrent to bank runs by insuring that the deposits (up to a limit) will be paid in full when a bank is closed by its charter authority. A charter authority is typically either the bank’s state banking commission or the Office of the Comptroller of the Currency (OCC). The charter authority will close a bank if the bank’s capital is deemed to be too low to be sustainable. Suppose a bank is believed to be insolvent when the capital that protecting deposits drops to a threshold as a specified percent (say, 2%) of asset value. The total capital is the sum of tier-1 and tier-2 capital, which is the sum of tangible equity and subordinated debt. This amounts to $[V - (D + D_1)] + D_1 = V - D$. Let $V_a$ be asset value at the time when charter authority closes the bank. Then, $V_a - D = 2\% V_a$, implies $V_a = D / 0.98$. In general, charter authority closes a bank when its asset value reaches $V_a = \kappa D$, where $\kappa \geq 1$.

The FDIC functions both as a receiver of the closed bank and an insurer of the deposits. As a receiver, the FDIC liquidates the assets of the closed bank in its best effort to pay back the bank’s creditors. Following the tradition in structural models, we assume that the insurance corporation’s liquidation cost, $\beta V_a$, is proportional to the asset value $V_a$ when the bank is closed. It is possible that the insurance corporation’s liquidation costs are different from the costs of liquidating the bank through the bankruptcy procedures, and thus we admit $\beta \neq \alpha$. Without going through bankruptcy court, it is indeed likely that $\beta < \alpha$.

As an insurer, the FDIC pays $D$ to depositors. Consequently, the insurance corporation lose $D - (1 - \beta)V_a$ if it is a positive number and 0 otherwise. This loss function can be written as $[D - (1 - \beta)V_a]^+$, where $[\cdot]^+$ returns only the positive part of its argument. Since $V_a = \kappa D$, the loss function is positive if $\kappa < 1 / (1 - \beta)$, in which the insurance corporation expects to suffer a loss after a bank closure. To cover the loss, the FDIC charges insurance premium on banks. In 2006, Congress passed reforms that permitted the FDIC to charge banks risk-based premium. It also allowed FDIC some authority to manage the Deposit Insurance Fund (DIF) into which the premiums are invested. At present, for deposit insurance assessment purposes, an insured depository institution is placed into one of four risk categories each quarter, determined primarily by the institution’s capital levels.

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8Our analysis actually works for the case of $V_a > D / (1 - \alpha)$. A structure where $V_a > D / (1 - \alpha)$ can be interpreted as a situation in which depositors are risk averse or demand cash immediately for their liquidity needs.

9This belief was reflected during the credit crisis, when the FDIC deposit insurance limit was raised from $100000 to $250000 on October 3, 2008.

10That is, $[x]^+ = x$ if $x > 0$ and $[x]^+ = 0$ if $x \leq 0$.

11In practice, the FDIC expects a loss because liquidation cost is uncertain. To keep the analysis tractable, we assume a fixed $\beta$ instead. Under a fixed cost $\beta$, the FDIC expects a loss if and only if $(1 - \beta)V_a < D$. Then, in view of $V_a = \kappa D$, the FDIC expects a loss if and only if $\kappa < 1 / (1 - \beta)$.
and supervisory evaluation. Hence a riskier bank pays higher insurance premium than a less risky bank. Recall that $I$ denotes the total deposit insurance premium a bank pays.\footnote{Until 2010, the FDIC assess the insurance premium based on total deposits. The assessment rate of the insurance is $a$ such that $I = aD$. However, there have long been concerns that banks shift deposits out of account temporally at quarter-ends to lower the assessment base. As required by the Dodd-Frank Act (Section 331), the FDIC have changed the assessment base to the difference between the risk-weighted assets and tangible equity since April 2011. If $V$ equals the risk-weighted assets, the new assessment base equals $D + D_1$, which implies the new assessment rate is $b$ such that $I = b(D + D_1)$. The actual premium calculations may also depend on the credit ratings and the proportion of long-term debt to deposits. See Federal Register, Vol. 76, No. 38, Friday, February 25, 2011, Rules and Regulations, for more details.}

The economic role of FDIC and charter authority in a full information model presented in the paper wherein the depositors can run at the right time to make their deposits risk-free can be explained as follows: the charter authority will close the bank at a later point than the depositors choose to run were there no deposit insurance or regulation. The FDIC increases the expected life of the bank. In turn, the bank will pay the insurance premium to FDIC in “good states” when it is solvent. This transfer of payments across the states can improve the overall value of the bank by increasing the present value of the tax shields and reducing the expected costs of default. With taxes and costs associated with bankruptcy, our analysis will show that the combined actions of the charter authority and the FDIC coupled with these important market frictions creates additional value to the bank, and lowers the dead-weight losses associated with bankruptcy.

Equity holders can potentially choose to default debt, which consists of deposits and subordinated debt, before bank run or closure. Absent bank run and closure, there is an optimal point for equity holders to default. This default decision maximizes equity value, given a liability structure. The optimal default of debt was referred to as endogenous default and derived by Leland (1994) for firms with long-term debt but without deposits. In the presence of deposits, we can derive the point of endogenous default. Let $V_d$ be the critical point for endogenous default, i.e., equity holders choose to default if and only if asset value $V$ reaches to or below $V_d$, if there were no bank run or closure. Then bankruptcy happens if either equity holders endogenously choose to default debt or the bank is closed due to bank run or by charter authority. Let $V_b$ be the critical point for bankruptcy, then $V_b = \max\{V_d, V_a\}$.

In summary, there can be three types of bankruptcy. The first type is endogenous default chosen by equity holders before a bank is closed due to bank run or by charter authority. In this case, liquidation of assets have to go through bankruptcy court and bankruptcy cost is $\alpha V_d$. The second type is bank run, which happens before endogenous default. Bankruptcy cost in the case of bank run is $\alpha V_a$, as it goes through bankruptcy court. The last type is bank closure by charter authority under FDIC insurance and regulation. In this case, bankruptcy cost is $\beta V_a$. Concisely, we denote bankruptcy cost by $\phi V_b$, where $\phi$ equals $\alpha$ or $\beta$ depending on the type of bankruptcy. At bankruptcy when the bank’s assets are liquidated, depositors are paid first, and the subordinated debt holders are paid the next if there is
value left over for them. Consequently, the payoff to debt holders is \([(1 - \phi)V_b - D]^+\).

In order to control the likelihood of bankruptcy, banks around the world are subject to equity requirements which form the core part of the international accord, commonly referred to as the Basel III, agreed in 2011 by bank regulators. In Basel III, all banks are required to hold core common equity above 7 percent of the risk-weighted assets. This is a substantial increase from a 4-percent requirement in Basel II. For large banks that are identified by regulators as “systemically important,” the core common equity requirement can be 0~2.5 percent higher. In our model, core common equity is \(V - (D + D_1)\) and the value of risk-weighted assets is \(V\). If the equity requirement is \(\zeta\), a bank must choose a capital structure that satisfies \([V - (D + D_1)]/V \geq \zeta\). This imposes a limit on leverage because it implies \(D + D_1 \leq (1 - \zeta)V\). If a bank’s capital structure does not meet the equity requirement, its regulator instructs the bank to adjust its capital structure, but the regulator will not close the bank if the bank is not insolvent. Therefore, we model equity requirement as constraint on banks’ choice of liability structure.

### 3 Valuation and Optimization

Before proceeding with bank valuation, it is useful to summarize the exogenous parameters in our model and our assumptions on the parameters. The parameters and assumptions are listed in Table 1. The third column presents our assumption on the range of values for the parameters. Particularly, we assume account service income is positive but doesn’t cover the entire cost of taking deposits: \(0 < \eta < r\). We also assume the existence of tax: \(0 < \tau < 1\). The bankruptcy and FDIC liquidation are both costly in our model: \(0 < \phi < 1\) and \(0 < \beta < 1\). We believe that these assumptions are realistic, and they are the requisite mathematical conditions to carry out valuation and optimization.

#### 3.1 Bank Valuation and Endogenous Insurance Premium

Since deposits are either withdrawn before bank run or FDIC insured, it is a safe security. Consequently, the value of deposits is its par value \(D\). Since the net interest payment on deposits amounts to \(C = (r - \eta)D\), which is the bank liability on deposits, the value of deposits is related to deposit liability by \(D = C/(r - \eta)\).

The value of subordinated debt and equity are affected by the risk of default, leading to bankruptcy. Consequently, the Arrow-Debreu price of bankruptcy plays a key role in bank valuation. Consider a security that pays $1 when default occurs, and 0 otherwise. The price of this security is the Arrow-Debreu price of bankruptcy, which is also the risk-neutral probability of bankruptcy. It is well known that the state price \(P_b = [V_b/V]^{\lambda}\), where \(\lambda\) is an increasing function of \(r\) and decreasing function of \(\delta\) and \(\sigma\). If cash flow of the assets is zero, \(\delta \equiv 0\), we have \(\lambda = 2r/\sigma^2\), which is proportional to \(r\) and inversely proportional to
The exact function of \( \lambda \) is given in equation (21) in Appendix. The state price \( P_b \) can be derived as solution to Merton’s (1974) no-arbitrage pricing equation (20) and \( \lambda \) is the positive root of a quadratic equation (22). The details of the derivation can be found in Appendix.

Bank value depends on its liability structure \((I, C, C_1)\) because the liability structure affects bankruptcy boundary and its state price. The following theorem summarizes the precise relation between bank value and liability structure.

**Theorem 1** Given liability structure \((I, C, C_1)\), bank run/closure boundary is \( V_a = \kappa C / (r - \eta) \), endogenous default boundary is

\[
V_d = (1 - \tau)[\lambda/(1 + \lambda)](I + C + C_1)/r,
\]

and bankruptcy boundary is \( V_b = \max\{V_a, V_d\} \). The equity, subordinated debt, and bank values are

\[
E = V - (1 - P_b)(1 - \tau)(I + C + C_1)/r - P_b V_b
\]

\[
D_1 = (1 - P_b)C_1/r + P_b[(1 - \phi)V_b - D]^+
\]

\[
F = V + (1 - P_b)\left\{(r + \eta/(r - \eta)][C + \tau C_1]/r
- (1 - P_b)(1 - \tau)I/r - P_b \min\{\phi V_b, V_b - D\}\right.
\]

The liability structure in the theorem includes insurance premium, but if we set \( I = 0 \), these formulas in the theorem also apply to banks without deposit insurance.

The endogenous credit spread of subordinated debt, which depends on the liability structure, can be obtained from Theorem 1. The endogenous credit spread is

\[
s = C_1/D_1 - r,
\]

where \( D_1 \) is a function of \( C_1 \) as given in equation (3). This credit spread takes the probability of bankruptcy into account through state price \( P_b \), which is affected by the bank liability structure. It is worth pointing out that the insurance premium also affects the credit spread, even though \( I \) does not appear in equation (3) explicitly. The insurance premium has an influence on endogenous default boundary in equation (1). Thus it affects bankruptcy boundary \( V_b \) and the state price of bankruptcy, through which it influences the credit spread.

Theorem 1 shows the role of account service income and deposit insurance in bank valuation. Along with tax savings on debt, account service income (positive \( \eta \)) increases the bank value as shown by the second term on the right-hand side of equation (4). The ability of the bank to attract deposits at a rate lower than the risk-free rate comes at a price: the bank has to close and incur bankruptcy cost if depositors run or charter authority closes the bank. The last term of the equation reflects the value lost due to bankruptcy. For a bank with deposit insurance, insurance premium also reduces the bank value, which is evidenced by the third term of the equation. Notice that even though insurance premium does not appear in equation (3), it affects the subordinated debt value through its influence on default boundary \( V_b \), on which debt value hinges.
A comparison of our bank valuation with Leland’s (1994) firm valuation is revealing. If we set \( C = 0 \) and \( I = 0 \) but \( C_1 > 0 \), all formulas in Theorem 1 reduce to those derived for the capital structure of firms with only debt—the word “subordinated” drops because there is no deposit in this case, and hence there is no subordination. If we set \( I = 0, \eta = 0, \) and \( C_1 = 0 \) but \( C > 0 \), the valuation formulas in Theorem 1 coincides with Leland’s case for debt protected at level \( \kappa D \). Leland’s seminal capital structure theory is for firms; the theory does not apply to banks that take deposits, earn account service income, pay deposit insurance premium, and subject to bank run or closure. Our theory extends Leland’s to banks and offers a consistent framework to understand the similarities and differences between banks and other firms.

Deposit insurance premium in Theorem 1 is exogenously given, but it should endogenously depend on the amount of deposits under insurance and the risk involved, among other things. In principle, the deposit insurance corporation should charge each bank the fair insurance premium, which makes the insurance contract worth zero. The next Theorem characterizes the fair insurance premium.

**Theorem 2** Given deposits \( D \), the fair insurance premium is

\[
I^o = r[1 - (1 - \beta)\kappa]^+DP_a/(1 - P_a),
\]

where \( P_a = \kappa D/V \) is the state price of bank closure.

An alternative way to write the insurance pricing equation is

\[
(1 - P_a)(I^o/r) = P_a[1 - (1 - \beta)\kappa]^+D,
\]

which says that the expected present value of the insurance premium paid to the insurance corporation equals the expected present value of the insurance obligations when the insured bank is closed by its charter authority.

The fair insurance premium, \( I^o \), increases with \( D \). Not only the insurance premium increases in deposits, the rate of insurance premium on each dollar of deposits also increases in \( D \). By Theorem 2, the rate of insurance premium is

\[
h = I^o/D = r[1 - (1 - \beta)\kappa]^+P_a/(1 - P_a).
\]

It is increasing in \( D \) because \( P_a \) is increasing in \( D \). This makes sense because expanding deposits increases the risk exposure of insurance corporation. If \( \kappa < 1/(1 - \beta) \), the fair premium \( I^o \) is positive. It converges to zero when \( \kappa \) rises to \( 1/(1 - \beta) \). If \( \kappa \geq 1/(1 - \beta) \), fair premium is zero because the bank will be closed with enough asset value to fully cover the losses of depositors.

Some academics have argued that FDIC has provided subsidized insurance to banks by charging a premium which is lower than its fair rate.\(^{13}\) To allow for potentially subsidized

\(^{13}\)See Duffie et al. (2003). On the other hand, it can be argued that the subsidy is necessary to compensate the insured banks for the costs of reporting requirements and tight regulation.
insurance premium, we assume that the subsidized FDIC insurance premium is \( I = \omega I^0 \), where \( \omega = 1 \) represents fair insurance premium and \( \omega < 1 \) represents the insurance subsidy. Relating to the net interest on deposits by \( D = C/(r - \eta) \), we have \( I = iC \), where

\[
i = \omega[1 - (1 - \beta)\kappa]^+ [r/(r - \eta)]P_a/(1 - P_a) .
\]

(8)

If the insurance corporation gives an insurance subsidy, then such a subsidy increases the bank value because the bank pays lower insurance premium while receiving the risk-free value of deposits.

With endogenous insurance premium, liability structure is characterized by the pair \((C, C_1)\) because \( C \) determines \( I \). Imposing endogenous insurance premium \( I = iC \) in bank value formula (4), it is straightforward to obtain

\[
F = V + (1 - P_b)[\tau(1 + i) + \eta/(r - \eta)] - iC/r
+ (1 - P_b)\tau C_1/(r - P_b) \min\{\phi V_b, V_b - D\} .
\]

(9)

On the right-hand side of equation (9), the second term is the value of tax deduction and account service income, netted off against the insurance premium that the bank pays for its deposits. The third term is the value of tax benefits to the bank for its interest expense on subordinated debt. The last term is the loss of bank value due to bankruptcy, for which bankruptcy cost \( \phi \) takes the value of \( \alpha \) or \( \beta \), depending on the type of bankruptcy.

3.2 Optimal Liability Structure

Now we proceed to examine how a value-maximizing bank will choose its liability structure, but we first consider the case of a regulated bank without FDIC deposit insurance. In this case, the bank has to take into account the potential for the depositors to run at any time to keep their loans risk-free. This “unregulated market” benchmark serves as a useful counter-factual for evaluating the effects of adding regulatory mandates such as the FDIC insurance, charter authority to close troubled banks, and capital adequacy requirements. This allows us to examine how a bank might rearrange its liability structure when different regulatory tools are mandated by the regulators. Later, we are able to gain some insights into the “optimal response” of a bank to regulatory mandates. For example, we will be able to make both qualitative and quantitative statements about the economic implications of the actions taken by the charter authority and the FDIC on the optimal response of the bank is adjusting their leverage, liability structure, and default decisions, relative to the counter-factual in which the banks make value-maximizing choices, unfettered by any government interventions.

As pointed out earlier, liability structure of an unregulated bank is completely described by the pair \((C, C_1)\). An optimal liability structure is the deposit liability \( C^* \) and subordinated debt liability \( C_1^* \) that maximizes bank value. The theorem below provides
Theorem 3 The optimal liability structure of an unregulated bank is unique. In the optimal liability structure, $V_a^* = V_d^*$, and the state price of bankruptcy $P_b^*$ is given in equation (41). The optimal deposits and subordinated debt liabilities are

$$C^* = (r - \eta)V P_b^{*1/\lambda}(1 - \alpha)$$

(10)

$$C_1^* = rVP_b^{*1/\lambda} \left[ \frac{1 + \lambda}{(1 - \tau)\lambda} - \frac{r - \eta}{r/(1 - \alpha)} \right].$$

(11)

This theorem is a direct consequence of Lemma 1 in Appendix. Equation (41) in Lemma 1 gives the exact formula of $P_b^*$; it is an elementary algebraic function of the following exogenous parameters: $r, \sigma, \delta, \tau, \eta,$ and $\alpha$.

The theorem characterizes the optimal liability structure of a bank that faces taxes, bankruptcy cost, and issues deposits at a rate lower than the risk-free rate. Combining Theorem 3 with Theorem 1, we can obtain analytical solutions for deposits $D^*$, subordinated debt $D_1^*$, equity $E^*$, bank value $F^*$, bankruptcy boundary $V_b^*$, and credit spread $s^*$ in the optimal capital structure of unregulated banks without FDIC deposit insurance. The optimal structure gives an optimal ratio $x^* = C_1^*/C^*$ of the subordinated debt liability to deposit liability. Equation (40) in Appendix presents the formula of the optimal liability ratio in terms of exogenous parameters. Whenever the liability ratio $x$ is given along with the deposit liability $C$, the subordinated debt liability can be calculated as $C_1 = xC$.

A key insight of the theorem is that it is optimal for banks to choose the precise amount of subordinated debt such that endogenous default boundary coincides with bank-run boundary. While this is proved mathematically in Appendix, we provide below the economic reasoning for this result, which is intuitive. For a bank, a special benefit of taking deposits is the discounted interest rate that the bank gets for providing account services, in addition to tax savings, whereas the cost of taking deposits is the loss associated with a bank-run. In contrast, subordinated debt brings tax savings but produces no account service income, while its cost is the loss associated with bankruptcy. Therefore, at the margin, the bank should use deposits, not debt, to balance the tax and liquidity benefits with the cost of bankruptcy. With this balance, the bank should take as much debt as possible for availing the tax benefits but should avoid the costs of bankruptcy resulting from endogenous default. To avoid the cost associated with endogenous default, the bank should avoid setting the endogenous default boundary above the bank-run boundary. As a result, the optimal subordinated debt should be set so that default occurs at exactly the same point as the bank-run.

This result has implications on the debate over whether subordinated debt provides a market discipline and in a credible way provides an early signal about the bank’s default
risk. Flannery and Serescu (1996) and Gorton and Santomero (1990) have addressed this question. Our result shows that the bank will endogenously adjust its subordinated debt so as to make the default boundary coincide with the run boundary. In this case, the bank essentially cancels out any signaling benefits that the regulators may be hoping for by requiring the bank to hold subordinated debt. This endogeneity is important be to addressed in empirical literature on this issue.

The assumption rational bank run, $\kappa = 1/(1 - \alpha)$, is essential for Theorem 3 to hold. In fact, it can be proved that the unique optimal liability structure can lead to $V_d^* > V_a$ if $\kappa < 1/(1 - \alpha)$, which would imply that the depositors run from the bank at a point when the bank would not have enough assets to refund the deposits in full after bankruptcy proceedings. Such a late bank-run is clearly not rational if depositors know the bank asset value.

Account service associated with deposits is very important for Theorem 3. In fact, if all assumptions of the theorem hold except that $\eta = 0$, then it can be shown that for every liability ratio $x \geq x^*$ there exists an optimal structure, and setting $\eta = 0$ in the theorem’s formulas gives the optimal structure with the largest amount of deposits. We can think of these optimal structures for a firm that faces tax but does not receive account service income. In this case, the optimal liability structure is not unique. A lower liability ratio in the optimal structure corresponds to more deposits. The optimal capital structure is not unique because deposits and subordinated debt have the same tax benefits and bankruptcy cost in the absence of account service. This suggests that a structural model without considering bank account service is not appropriate for understanding bank decision about its optimal combination of deposits and subordinated debt.

Taxes play an important role in Theorem 3. If all assumptions of the theorem hold except $\tau = 0$, it can be shown that for every liability ratio $x \leq x^*$ there exists an optimal structure. Setting $\tau = 0$ in the theorem gives the optimal structure with the smallest amount of deposits. This special case corresponds to a bank which account service but receives no tax benefit. In fact, this is the typical setup of banks in the banking literature that focus on the role of bank account services. Without tax benefit, banks have no incentive to issue subordinated debt because it exposes banks to bankruptcy cost. In this situation, as long as debt level is low enough so that default never happens before bank run, default cost is irrelevant to bank valuation. Therefore, as long as the liability ratio sets default boundary below bank-run boundary, any liability structure with subordinated debt is optimal. This indeterminacy of subordinated debt in the absence of tax benefit suggests that a model of a bank without consideration of tax savings is not appropriate for understanding bank

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14 Without account service income, deposits in our model bears resemblance to secured debt in Leland’s (1994) because deposits is protected by bank run. Leland (1994) considers the optimal capital structure of firms that take either secured or unsecured debt but not the optimal mix of the two. Leland analytically solves the optimal capital structure of firms that take unsecured debt, but for firms that takes secured debt, he solves their optimal structure numerically.
optimal liability structure.

Theorem 3 ignores deposit insurance and FDIC regulation of banks, but it will serve as a useful counterfactual case to examine the effects of deposit insurance and FDIC regulation. If we consider banks under FDIC, the optimal liability structure is a pair of $C^*$ and $C_1^*$ that maximizes the bank value in equation (9) subject to equation (8). The value-maximizing bank in our framework is fully aware that any decision pertaining to leverage and liability structure will have a consequence on the FDIC insurance premium. It will therefore be very mindful of this channel in its choice of leverage and liability structure. The endogenous determination of FDIC premium, leverage and the liability structure allows us to capture the feedback channel from FDIC to the banks and vice versa.

The next theorem, which is a direct consequence of Lemma 2, characterizes the conditions for a liability structure to be optimal. The theorem first presents a necessary condition. The condition is also sufficient if the closure rule is ironclad enough or the FDIC subsidy is generous enough to keep the insurance premium low relative to the account service income.

**Theorem 4** Consider a bank that has deposit insurance and regulated by FDIC and its charter authority. The FDIC insurance premium is endogenous, and bank closure rule is $V_a = \kappa D$, where $\kappa \geq 1$. The liability structure $(C^*, C_1^*)$ of the bank is optimal if only if endogenous default boundary is not below bank closure boundary, i.e., $V_d^* \geq V_a^*$. In the optimal structure, the state price of bankruptcy $P_b^*$ satisfies equation (42), and the optimal deposits and subordinated debt liabilities are

\[
C^* = (r - \eta)VP_b^{1/\lambda}/\kappa
\]

\[
C_1^* = rVP_b^{1/\lambda}\left(1 + \lambda \frac{1}{(1 - \tau)\kappa} - \frac{r - \eta}{r\kappa} - \omega[1/\kappa - (1 - \beta)]^+ \frac{P_b^*}{1 - P_b^*}\right).
\]

The first part of theorem gives a necessary conditions for the optimal structure, which imposes a lower bound on optimal debt liability relative to deposit liability. A level of debt liability below this bound cannot be optimal. In particular, the theorem shows that zero subordinated debt $C_1 = 0$ is not optimal for a bank as long as insurance premium is not too high. If $C_1 = 0$, since insurance premium is $I = iC$, it follows from Theorem 1 that

\[
V_a = \kappa C/(r - \eta)
\]

\[
V_d = (1 - \tau)[\lambda/(1 + \lambda)](1 + i)C/r .
\]

Then, $V_a > V_d$, meaning the structure is suboptimal, if $i$ is very large.

Theorem 4 shows that facing high critical point for bank closure or large insurance premium subsidy relative to account service income, it is optimal for banks to leverage so that endogenous default of debt will be as late as the closure of banks. This is always the case if $\kappa \geq 1$. In fact, if $\kappa$ is substantially bellow 1, the liability structure might be optimal.
with $V_d > V_a$. We have confirmed this by both mathematical derivations and numerical optimization. However, the case of $\kappa < 1$ is not economically meaningful if the FDIC and charter authority closes a bank to protect deposits.

If $V_a = V_d$ is the best for banks, we can analytically derive the optimal liability structure. The exact formula of $P_b^*$ is given by equation (42) of Lemma 2 in appendix. It is a function of the following exogenous parameters: $r$, $\sigma$, $\delta$, $\tau$, $\eta$, $\beta$, $\kappa$, and $\omega$. Combining this theorem with Theorem 1 and Theorem 2, we obtain analytical solutions for the deposits $D^*$, subordinated debt $D_1^*$, equity $E^*$, bank value $F^*$, bankruptcy boundary $V_b^*$, credit spread $s^*$, and insurance premium $I^*$ in the optimal capital structure of a bank under FDIC regulation.

The most distinctive feature of Theorem 4 is optimization with endogenous insurance premium. Besides considering the tradeoff among tax benefit, account service income, bank closure, and bankruptcy cost, banks in this theorem also consider insurance premium. If we set $\omega = 0$ and $\kappa = 1/(1 - \alpha)$, the formulas in this theorem reduce to those in Theorem 3. With positive $\omega$ and more general $\kappa$ in this theorem, the insurance premium rate $\omega h$ is an increasing function in $D$, which causes $i$ to be increasing in $C$. Therefore, banks have to consider increase in insurance premium caused by $D$ and by the associated increase in the rate. In later part of the paper, we will show the impact of endogenous insurance premium on banks’ optimal choice of capital structure.

Optimization in Theorem 4 ignores equity requirement. Subject to equity requirement, banks have to maximize bank value $F$ by choosing $(C, C_1)$ subject to constraint $D + D_1 \leq (1 - \zeta)V$, where $D = C/(r - \eta)$ and $D_1$ is a function of $C_1$ defined by equation (3). The sum of deposits and subordinated debt is the total debt of the bank, and its ratio to total assets measures the bank’s leverage, which is referred to as leverage ratio: $l = (D + D_1)/V$. Equity requirement imposes a limit on leverage ratio: $l \leq 1 - \zeta$.

If an optimal capital structure in Theorem 3 or 4 happen to meet equity requirement, i.e., $D^* + D_1^* \leq (1 - \zeta)V$, then, the optimal liability structures in those theorems are also the optimal structures for banks subject to equity requirement. In this case, the equity requirement is not binding. Therefore, all the features of the optimal structures in those theorems remain when equity requirement is not binding. However, we lost some features when the requirement is binding, as summarized in the next two theorems.

For a bank that is not insured but subject equity requirement, the optimal liability structure is characterized as follows.

**Theorem 5** Assume a bank is not insured or regulated by FDIC but subject to equity requirement. There is a unique optimal liability structure that maximizes the bank value subject to the constraint of equity requirement. If equity requirement is binding, the optimal liability structure $(C^\dagger, C_1^\dagger)$ of the bank sets $V_a^\dagger \geq V_d^\dagger$. In the optimal structure, the state price of bankruptcy $P_{b^\dagger}$ is given by equation (48) for the case of $V_a^\dagger > V_d^\dagger$ and by equation...
(49) for the case of $V_{a}^{\dagger} = V_{c}^{\dagger}$, and the optimal liabilities are

\[
C_{\dagger} = (r - \eta)VP_{b}^{1/\lambda}(1 - \alpha) \quad (16)
\]

\[
C_{1}^{\dagger} = rV[1 - \zeta - (1 - \alpha)P_{b}^{1/\lambda}] / (1 - P_{b}^{1/\lambda}) \quad \text{if} \quad V_{a}^{\dagger} > V_{d}^{\dagger} \quad (17)
\]

\[
C_{1}^{\dagger} = rVP_{b}^{1/\lambda}[1 + \lambda] / (1 - \tau) \lambda \quad \text{if} \quad V_{a}^{\dagger} = V_{d}^{\dagger}. \quad (18)
\]

This theorem is a direct consequence of Lemma 3 in Appendix. Equations (48) and (49) of the lemma give the exact formulas of the state price of bankruptcy. In the formulas, the state price in the optimal structure is a function of following exogenous parameters: $r$, $\sigma$, $\delta$, $\tau$, $\eta$, $\alpha$, and $\zeta$.

The most important effect of a binding equity requirement is to push endogenous default boundary $V_{d}^{\dagger}$ potentially below bank closure boundary $V_{a}^{\dagger}$. This means that an optimal response to a binding equity requirement is associated with a reduction in subordinated debt. When endogenous default boundary is below bank run boundary, subordinated debt is at a level so that tax benefit is not fully exploited by the bank. However, this helps in meeting the equity requirement and taking advantage of deposit account service income. It is still possible that the optimal structure sets $V_{d}^{\dagger}$ at the same level of $V_{a}^{\dagger}$, in which cases the bank reduce both deposits and subordinated debt proportionally. In either case, one unvarnished truth in the optimal structure is that subordinated debt does not protect deposits.

**Theorem 6** [This is still a work in progress.]

### 4 Quantitative Analysis of Bank Liability Structures

The model developed in the preceding sections paves the way to examine quantitatively the optimal responses of bank liability structure to changes of economic and regulatory environment. In this section, we numerically investigate the optimal liability structure of banks operating in alternative economic and regulatory environments. The first type of banks we consider are those operate in freely competitive market as specified in Theorem 3, in which deposits are not insured and capital is not subject to constraints. The case of these banks serves as a counterfactual for FDIC insurance and capital requirements. The second type of banks in our quantitative investigation are those which operate under FDIC insurance and regulation as described in Theorem 4. The case of these banks allows us to isolate the effect of FDIC from capital requirement. The third type of banks we inspect operate without FDIC insurance but are subject to capital requirement as characterized in Theorem 5. Banks in Europe, where governments do not provide deposit insurance, are of this type since European banks are subject to capital requirements. In the U.S., there are
also banks uninsured by the FDIC but subject to capital requirements. The last type of banks we study are are the majority of banks in the U.S.; they are insured by the FDIC and subject to capital requirement as in Theorem 6.

### 4.1 Exogenous Parameters and Endogenous Variables

Our model inherits the major advantage of structural models that coherently connects the risk of debt and equity to the risk of assets. Asset volatility ($\sigma$) is an important parameter in classic structural models. However, accounting filings and market price do not show asset volatility of a bank directly, and thus people have to infer asset volatility from accounting data and market prices. Moody’s KMV provides estimates of asset volatility for a large number of companies across a large range of industries. In Figure 2 we present the average, median, and the 10/90-percentiles of Moody’s estimates of asset volatilities for banks (panel A) for 2001–2012. As a comparison, we also present the estimates for manufacturing firms (panel B). The figure demonstrates how bank assets are different from the asset in manufacturing firms: bank assets have much lower volatility. The average bank asset volatility is around 10 percent. In contrast, the average asset volatility of manufacturing firms is between 40 and 50 percent. Although bank asset volatility fluctuates over time, the median is around 5 percent for the entire time, and the 90 percentile is well below 15 percent for 2001–2007. Even for the turmoil period of 2007–2012, the 90 percentile of bank asset volatility stays well below 25 percent. In view of this Figure, we use $\sigma = 10\%$ as the baseline value for asset volatility, as shown in Table 1, but explore alternative values in a range between 5 and 25 percent.

Another important parameter of bank assets is its rate of cash flow ($\delta$). In the risk neutral world, the growth rate of asset value is $r - \delta$. We focus on the steady-state optimal structure with perpetual debt liability and thus assume that the assets are in a steady state with zero growth rate. This implies $\delta = r$. Otherwise, the constant liability appears decreasing relative to asset value, which implies decreasing risk of bankruptcy over time. To compare the risk of bankruptcy and leverage across banks, it is common to measure each component of the capital structure as a percent of asset value. We normalize bank asset value to 1. To think of this in the context of a practical bank, one may regard the bank asset value as $V = \$1 billion$.

As a prime benefit of taking deposits, we cannot ignore the income from deposit account service when considering bank optimal liability structure. In principle, the net income from deposit account service should be determined in the competitive market of deposits. In a perfect competitive market, the net income would be driven to zero in an unregulated banking industry or covers the insurance premium in an industry with deposit insurance. However, new entry of banks into the market is regulated by charter authorities. With limit of entry, deposit account service should be profitable, and the profitability should depend on the amount of deposits and the bank. Thus, the parameter $\eta$ should differ across banks...
and should be a function of $D$. However, we do not explicitly model the market equilibrium of deposits or the dependence of $\eta$ on $D$.\footnote{While modeling the market equilibrium of deposits is well beyond the scope of this paper, it is possible to assume that $\eta$ is a function of $D$ and then determine the function’s parameters in calibration.} in order to keep the model tractable and focus on the effects of regulations, we instead assume $\eta$ to be constant but allow it to be different for different banks. We will first assume a range of values for $\eta$ in the basic numerical analysis and later estimate $\eta$ from data in calibrations. For the baseline number, we set $\eta$ to be 50 basis points, based on the average service charges on deposit accounts of the 50 largest commercial banks.

A major reason for leverage is the ability to deduct tax, and thus corporate tax rate is an important factor to determine capital structure. The statutory corporate tax rate in the U.S. ranges up to 35 percent, one of the highest in the world. The U.S. Treasury Department estimated in 2007 that the effective marginal tax rate on new investment in business is 25.5 percent, but it varies substantially by business sectors. The actual tax benefit of debt varies across firms. The academic literature suggests that the effective corporate tax rate is around 10 percent for non-financial firms (Graham, 2000) but more than 30 percent for banks (Heckemeyer and Mooij, 2013). We choose $\tau = 30\%$ as the baseline value for corporate tax rate and later vary it to examine the sensitivity of optimal structure to the rate.

Countering the tax benefit of debt is the bankruptcy cost, and measuring it is a challenging task. A well-known reference is the study of Altman (1984), which examines a sample of 19 industrial firms which went bankrupt over the period of 1970–1978. The study concludes that the bankruptcy cost is estimated as 19.7\% of a firm value just prior to its bankruptcy. However, Bris, Welch and Zhu (2006) show that bankruptcy costs are heterogeneous across firms and types of bankruptcy. The estimation of the costs is also sensitive to the methodology. They conclude that the total bankruptcy cost ranges between 0\% and 20\% of firm assets. Bankruptcy costs of banks are found to be higher. Based on 791 FDIC-regulated commercial banks failed during 1982–1988 (the savings and loan crisis) James (1991) estimates that bankruptcy cost is 30\% of a failed bank’s assets. Based on 325 insured depository institutions failed during 2008–2010 (the Great Recession), Flannary (2011) estimates that bankruptcy cost is 26.6\% of a failed bank’s assets. In light of these estimates, we set $\alpha = \beta = 25\%$ as the baseline value.

We choose the baseline values of other exogenous parameters in the following way. The starting assumption about bank closure boundary is 102\% of deposits. Generally, a federal or state banking regulatory agency closes a bank when it is unable to meet its obligations to depositors. Therefore, the closure parameter $\kappa$ should be at least as large as 1. When a bank’s total capital is less than 2\% of the assets, the FDIC classifies it as “critically undercapitalized,” and the charter authority typically closes the bank. This leads to $\kappa = 102\%$. For insurance subsidy, we use $\omega = 1$ (no subsidy) as the starting point in
view of the 2011 FDIC reform but later examine the effects of subsidized deposit insurance. For capital requirement, we use the 7-percent tier-1 ratio in Basel III as the baseline value. That is, $\zeta = 7\%$. All the baseline values are listed in Table 1.

Given the exogenous parameters, a bank that maximizes its value chooses the liability structure. The chosen liability structure can be characterized by a set of variables endogenously determined by the value-maximizing management and the valuation in the market. Table 2 lists the endogenous variables of our interest and presents the mathematical definition of our model. The first endogenous variable is the charter value of the bank, which is the difference between the bank value and asset value and expressed as percent of the asset value. The next three endogenous variables are the deposit, subordinated debt, and equity ratios to asset value. The sum of deposit and subordinated debt ratios equals the leverage ratio, the ratio of the total debt to total asset. The level of deposits controls the closure boundary, and the choice leverage controls the endogenous default boundary. These two boundaries are two endogenous variables of our interests. The higher of the two is the boundary of bankruptcy, which causes a loss in bank value due to bankruptcy cost, another endogenous variable of our interest. The possibility of bankruptcy causes the bank to pay a credit spread on the unprotected subordinated debt. The credit spread is an important endogenous variable that is observable in the market. For banks with deposit insurance, the insurance premium is endogenously determined and is of our interest.

4.2 High Leverage of Unregulated Banks

In Table 2, numerical values of the endogenous variables are calculated, based on the analytical solutions in Theorems 3 and 4 for both unregulated banks and FDIC-insured banks without subjecting to equity requirement. Unregulated banks are rarely found in the modern world, but consideration of this type helps us to answer an important question: are banks fundamentally different from non-financial firms regardless the regulations imposed on banks? Answering this question allows us evaluate the effects of deposit insurance and capital requirement. FDIC-insured banks without capital requirement do not exist in modern world either, but consideration of this type allows us to separate the effects of deposit insurance from the effects of ever-changing capital requirement. We will finally examine banks subject to capital requirements.

The numerical values in Table 2 reveal some special characteristics of bank optimal liability structure. The most distinctive characteristic is the high level of leverage we do not see in non-financial firms but are typical in banks. It is important to note that the leverage is high without deposit insurance. In an unregulated bank, the sum of deposits and subordinated debt amounts to 94.58 percent of the asset value, leaving only 5.42 percent of tangible equity.

The cause of the high leverage must be due to one or both of the two fundamental factors that distinguish unregulated banks from other firms: (1) taking deposits to provide account
services and (2) earn cash flows from low-volatility assets like loans. We examine these two factors further in Table 3, in which we consider a range of asset volatility and account service income rate. It is not surprising that for higher volatility the optimal structure consists of fewer deposits and less subordinated debt, and thus is lower leveraged. What is especially revealing is the sensitivity of leverage to volatility, as shown in panel A. For a moderate asset volatility of 25 percent, the leverage ratio sharply reduces to 76.90 percent, giving the bank 24.10 percent tangible equity.

Another notable characteristic of the optimal liability structure presented in Table 2 is the significant amount of subordinated debt. The subordinated debt in an unregulated bank is 56.97 percent. The reason for subordinated debt is that banks optimally set the subordinated debt to the level such that the endogenous default boundary is the exactly the same as the closure boundary, as predicted by Theorems 3. For unregulated banks, closure and default boundaries are the same in Table 2 and in panel A of Table 3. With this debt strategy, banks maximize the benefit of tax deduction without affecting the probability of bankruptcy. The benefit of tax deduction goes into the charter value of the bank.

Besides low asset volatility, account service income is a cause of bank leverage, and the effects are shown in panel B of Table 3. It is not surprising that higher account service income on deposits induces more deposits in the optimal capital structure. If we interpret the account service income as the premium on providing liquid cash accounts, this is consistent with DeAngelo and Stulz (2013), who argue that premium on liquidity production is a reason for high leverage in banks with deposits. A new perspective shown in panel B of Table 3 is that higher account service income (or liquidity premium) on deposits also causes more leverage of subordinated debt. The higher level of deposit raises the closure boundary and then allows higher level of subordinated debt to take advantage of additional tax benefit.

The high leverage causes a loss in bank value due to bankruptcy. For an unregulated bank in Table 2, the lost value is 3.15 percent of the asset value. For the potential bankruptcy loss that the highly leveraged bank, the subordinated debt holders demand 101 basis points of credit risk premium. The last two rows in panel A of Table 3 demonstrates that asset volatility is a major driver of bankruptcy loss and credit spread, even though it pushes down leverage. For an asset volatility as high as 25 percent, the credit spread is 3.28 bps, three times as high as the credit spread for 10% volatility. Higher asset volatility drives up the probability of bankruptcy and thus credit spread. The last row in panel B of Table 3 illustrates the positive relation of bankruptcy loss and credit spread to account service income. The relation is positive because higher account service leads to more deposits and subordinated debt in the capital structure.

For unregulated banks, optimal response completely changes the relation between bank closure boundary and bankruptcy cost. To demonstrate this, we first assume an unregulated bank optimizes its liability structure as presented in Table 2 and then alter the bankruptcy
cost $\alpha$. If the bank does not adjust its liability structure for the changed bankruptcy cost, we expect the bank closure (bank run) boundary to be higher (or lower) if bankruptcy cost increases (or decreases) because the closure boundary is $V_\alpha = \kappa D = D/(1 - \alpha)$. Such relation between $V_\alpha$ and $\alpha$ is plotted as the dashed line in Figure 3. Since subordinated debt is constant in the fixed liability structure, the endogenous default boundary is independent of $\alpha$ as marked by circles in the figure. However, when the bank optimally responds the change of bankruptcy cost, it changes deposits, resulting a decreasing relation between $V_\alpha^*$ and $\alpha$, which is plotted as the solid line. Since the optimal endogenous default boundary is the same as closure boundary, default boundary also decreases as $\alpha$ increases.

Therefore, when a liability structure optimally responses, the relation between bank closure/default boundary and bankruptcy cost is very different from the case of a fixed structure. Mathematically, $V_\alpha = D/(1 - \alpha)$ is an increasing function in $\alpha$ with fixed $D$, but $V_\alpha^* = D^*/(1 - \alpha)$ is in fact a decreasing function in $\alpha$. Panel A of Table 5 demonstrates that closure boundary of the unregulated bank is lowered by 1.26 percent of asset value when $\alpha$ rises from 25% to 30%. As a result of the optimal response, the value loss to bankruptcy gets larger only by 35 basis points of asset value when bankruptcy jumps up by 5 percent. Without optimal response, one would expect the credit spread to go up when bankruptcy cost goes up. To the contrary, the credit spread in the optimal structure goes down by 7 basis points. This is because the optimal structure cuts deposits down by 3.39 percent of asset value while boost subordinated debt up by 1.04 percent of asset value. The increase of subordinated debt associated with the increase of bankruptcy cost would have been difficult to understand without considering banks’ optimal response.

4.3 Effects of FDIC on Bank Liability Structure

Optimal responses of bank liability structure is very important in understanding the effects of changes in the environment banks operate. First of all, the optimal response affects the function of FDIC. The major purpose of FDIC is to reduce the probability of bank failure by preventing bank runs. This function allows banks to take more deposits while controlling the bank closure boundary below the potential bank-run boundary. As we have mentioned earlier, Table 2 shows that the bank closure boundary with FDIC insurance is 49.40 percent, slightly lower than the 50.15 percent closure boundary for unregulated banks, even though FDIC-insured banks take more deposits. The slightly lower closure boundary leads to slightly smaller loss of value to bankruptcy. In Table 2, the value loss due to bankruptcy for FDIC-insured banks is 3.01 percent of asset value, only slightly smaller than the 3.15 percent for unregulated banks. This also leads to a 4 basis point reduction in the credit spread. Judging by the small reduction of bankruptcy loss and credit spread, the effect of FDIC seems insignificant. The reason is that banks optimally response to FDIC insurance by increasing deposits, which attenuate the impact of FDIC on the probability of bankruptcy. The optimal response raises the bank value. In Table 2, the charter value of
a FDIC-insured bank is higher than the unregulated bank by 1.33 percent of asset value. Notice that the FDIC increases bank value even when it charges fair insurance, as assumed for the calculation in Table 2. The insurance premium is 11 basis points of asset value. The insurance premium is a cost to the bank, but the bank in Table 2 is able to benefit from FDIC insurance because of its optimal response, but the optimal response reduces the effect of FDIC on bank failures.

The optimal response of FDIC-insured banks changes the relation of its closure boundary to the rule of charter authority. In Figure 4, we first assume an insured bank optimizes its liability structure as presented in Table 2 and then change $\kappa$. If the bank keeps the liability structure fixed, the closure boundary is $V_a = \kappa D$, which is a linear function of $\kappa$ as the dashed line in the figure. The endogenous default boundary $V_d$ is independent of $\kappa$, as marked by circles. If the bank optimally responds to the change of $\kappa$, however, the relation of closure boundary with $\kappa$ is completely different; it does not increase much as $\kappa$ increases. Again, $V_a^* = \kappa D^*$ is not a linear function of $\kappa$ anymore because bank optimally reduced deposits in response to the increase of $\kappa$. The optimal default boundary $V_d^*$ relates to $\kappa$ the same way as $V_a^*$ because $V_d^* = V_a^*$ for all $\kappa$ plotted in the figure. The response of closure/default boundary to change $\kappa$ is complicated by the change of endogenous insurance premium. As $\kappa$ increases, the endogenous insurance premium decreases, as plotted in Figure 5. This reduces the cost of taking deposits and encourages banks to take more deposits in the optimal structure.

After accounting for banks’ optimal responses, deposit insurance still contributes to the high leverage of a bank’s optimal structure, at least partially. In Table 2, the leverage ratio of a deposit-insured bank is 95.17 percent, higher than the 94.58 percent leverage ratio of an unregulated bank. With deposit insurance, a bank holds more deposits but less subordinated debt. Intuitively, deposit insurance and the closure rule of charter authority allow an insured bank to take more deposits and keep the closure boundary lower than the boundary of depositors’ run in an unregulated bank. Imagining a bank switching from an unregulated to a deposit-insured in Table 2, the optimal deposits increases substantially from 37.61 percent to 48.43 percent. In the meantime, the closure boundary does not increase. Instead, it decreases slightly from 50.15 percent to 49.40 percent. This illustrates a major benefit bank received from the FDIC—it prevents bank from suffering from increased chance of bank run when increasing deposits.

As predicted by Theorem 4, the endogenous default boundary is always adjusted to the same level of closure boundary in FDIC-insured banks. If a bank switches from an unregulated to an FDIC-insured as in Table 2, the bank has to reduce subordinated debt from 56.97 percent to 46.74 percent in order to keep the default boundary the same as the closure boundary. Thus, FDIC insurance causes a reduction in subordinated debt. This is directly caused by the lower closure boundary due to charter authority. While the literature has noted that deposit insurance may cause banks to increase deposits, our model shows the effects of reducing subordinated debt. The reduction still leaves the bank with substantial
amount of subordinated debt, through which the bank takes advantage of the tax benefit.

The FDIC does not affect the dominant role of asset volatility in the high leverage of banks. Panel A of Table 4 shows that the optimal leverage is as sensitive to asset volatility in an FDIC-insured bank as in an unregulated bank. If asset volatility is as low as 5 percent, leverage ratio goes up to 111.16 percent, giving negative tangible equity. If asset volatility is 25 percent, leverage ratio reduces to 75.56 percent, giving 24.44 percent tangible equity. The effect of asset volatility on leverage shown in this table is similar to its effect shown in Table 3. Therefore, the importance of asset volatility as a factor of bank leverage is not diminished by FDIC insurance.

FDIC insurance does not change the role of account service in leverage either. In panel B of Table 4, leverage ratio is higher for higher account service income rate. If the account service income rate is lowered to 30 bps, the leverage ratio reduces to 92.23 percent. If the account service income rate is raised to 70 bps, the leverage ratio increases to 96.89 percent. Leverage ratio increases with the account service income because the deposits increase with it. The increased deposits raise the closure boundary, which gives more room for subordinated debt. As a result, the relation between leverage and account service income in an FDIC-insured bank in Table 4 is very similar to what we have seen in Table 3 for an unregulated bank.

In the above analysis, we assume that the FDIC liquidation cost is not different from the cost of bankruptcy in a private sector. However, there have been a view that the FDIC lowers the asset liquidation cost. For example, the U.S. lawmakers believe that the FDIC can implement an more efficient and orderly liquidation that protects the value of bank and thus grant the “orderly liquidation authority” to the FDIC by Title II of the Dodd-Frank Act.\textsuperscript{16} If this view is correct, the reduced liquidation cost may further affect the leverage of banks. Table 6 investigates this hypothesis. In this table, the FDIC liquidation cost $\beta$ is lowered from 25% to 20% while the private sector bankruptcy cost $\alpha$ is kept at 25%. As a result, both deposits and subordinated debt go up in the optimal liability structure. The leverage ratio increases by 2.05 percent. Without considering the optimal response of banks, one would expect bankruptcy loss and credit spread to reduce. To the contrary, bankruptcy loss goes up and the credit spread is up by 7 bps. The counterintuitive change of credit spread is due to the optimal increase of leverage.

4.4 The Role of Tax Policy in Bank Leverage

The benefit of tax deduction of is a major reason for leverage for all firms, not just for banks. Although the tax benefit of leverage is not a reason for banks to be different from other companies, corporate tax is more important for bank liability structure because banks take much higher leverage than non-financial firms. Observing the importance of tax benefit to

\textsuperscript{16}On the other hand, there is an view that the FDIC has no incentive to maximize liquidation value and thus results in larger loss of assets value (James, 1991).
banks, several recent papers have attempt to empirically measure the link between leverage and tax rate (see Heckemeyer and de Mooij, 2013, Keen, 2011, and Schandlbauer, 2013) based on linear regressions. In this area of research, our model offers a coherent framework for the link between tax rate and leverage. Based on our model, the optimal leverage of banks is lower in an economy with lower corporate tax rate. Moreover, the effects of corporate tax rate on bank leverage are almost the same for unregulated banks and FDIC-insured banks.

Table 7 shows the optimal responses to changes of corporate tax rate. If the corporate tax rate is lowered from 30% to 25%, both unregulated banks and FDIC-insured banks take less deposits and less subordinated debt. Especially, the banks reduce subordinated debt by more than five percent of the asset value. In comparison, the banks reduce deposits by only about one percent of the asset value. Banks react differently on deposits and subordinated debt for the following reason. The main purpose of subordinated debt is to take advantage of tax benefit. The reduction of subordinated debt gives up less benefit than a reduction of deposits because deposits earns income from account service. The reduction of deposits and subordinated debt lowers the leverage ratio by near seven percentage points in both the regulated bank and FDIC-insured bank in Table 7.

Because of the lessening of leverage, the value loss to bankruptcy and credit spread both drop if corporate tax is cut. This appears to support those who argue that lowering corporate tax for banks will stabilize the banks. However, if the lowering corporate tax rate leads to a loss in tax revenue, it is a costly policy change for the public to buy the stability of banks. An alternative is to lower corporate tax rate just for banks as suggested by Fleischer (2013). This certainly begs for the question of fairness of corporate tax and the question of potential distortion of the aggregate economy. These important issues are beyond the scope of this paper, but the link between tax and bank leverage should lay a stepping stone for a welfare analysis of the benefit and cost of tax policy reforms.

4.5 The Impact of Capital Requirements

The optimal response of banks to regulatory capital requirement is more complicated. Given the set of parameters in Table 1 for a representative bank, we calculate the optimal liability structure under various assumptions of capital requirement. We first consider an unregulated bank that is not subject to any regulator equity requirement. This is in fact the same bank in Table 2. We present its optimal values of the endogenous variables again in the second column of Table 8 for convenience of comparison with banks that are subject to an requirement. Notice that the leverage ratio without equity requirement is 94.58 percent, which implies a tangible equity ratio of 5.42 percent.

For the first case with capital requirement, we consider a bank that is not FDIC-insured but subject to a 4% regulatory equity requirement. This requirement is of particular interest because the minimum tier 1 capital requirement of Basel II is 4 percent of risk-weighted
assets (Basel Committee on Banking Supervision, 2004). Notice that the 94.58% leverage ratio without capital requirement meets the 4% equity requirement automatically. Obviously, there should be no response if a bank’s optimal liability structure automatically satisfies the requirement. The third column reports the optimal liability structure under the constraint of at least 4 percent tangible equity, as calculated according to Theorem 5. As expected, the optimal structure subject to the 4% equity requirement is identical to the structure without any requirement. This result supports the criticism that Basel II sets the capital requirement too low.

After the 2007–2009 financial crisis, Basel III raises the regulatory equity requirement to 7 percent of risk-weighted assets (Basel Committee of Banking Supervision, 2011a). Hirtle (2011a, 2011b) explains the rational for Basel to raise the requirement to 7 percent. The tangible equity ratio of 5.42 percent in the optimal capital structure without capital requirement does not meet the 7 percent requirement of Basel III. We calculate the optimal liability structure subject to 7% regulatory equity requirement and report the result in the fourth column of Table 8. The result shows that in order to meet a regulatory equity requirement of 7 percent, the representative bank must trim both deposits and subordinated debt. The leverage ratio reduces to exactly 93 percent to give 7 percent tangible equity. The restriction to leverage results in lower bankruptcy loss and credit spread, indicating a safer bank. The cost for the bank is that the leverage restriction lowers the bank value—the charter value is lower with binding capital requirement.

For large banks that are regarded systemically important by regulators, the new Basel rule imposes additional equity requirement, raising the total requirement up to 9.5 percent of risk-weighted assets (Basel Committee of Banking Supervision, 2011b). To comply with the rule, regulators in European Union and Switzerland have raised the regulatory equity requirement for large banks to 10 percent. For the parameters in Table 1, we calculated the optimal liability structure subject to a 10% regulatory equity requirement and present the result in the fifth column of Table 8. As expected, the leverage ratio reduces to exactly 90 percent to have 10 percent tangible equity. The higher capital requirement pushes the levels of deposits and subordinated debt further down. This makes the bankruptcy loss and credit spread even lower.

Admati et al. (2011, 2013) argue for even higher regulatory equity requirement. In light of their proposals, we present the optimal liability structures subject to 15 and 20 percent requirements. Again, banks take even less deposits and subordinated debt. The bankruptcy loss and credit spread drop significantly.

Banks pay for the safety brought by the regulatory equity requirement. In Table 8, the charter value decreases when the requirement increases. Interestingly, the stringent capital requirement does not force the bank to forfeit much of charter value. For example, the charter value drops only slightly, from 25.67 percent without requirement to 25.45 percent with 10% requirement. Even when the harsh 20% requirement is imposed, the charter value
still 24.01 percent of the asset value, which is only 1.66 percentage points lower than the charter value without requirement. A reason for the cost in bank value to moderate is compensation from the reduction in credit spread.

In all the cases we examined in Table 8, the closure boundary and default boundary are always same. It shows that the optimal reduction of leverage in response to equity requirement still avoid making the subordinated debt protect the deposits in a bank.

5 Conclusion

We conclude by briefly summarizing the issues that we have addressed in our paper, and by placing our work in the perspective of previous literature that is pertinent to our work.

Our paper provides a setting in which the following questions can be addressed. How much equity capital should a bank hold? What should be the ideal composition of insured deposits and long-term unsecured debt in the capital structure of banks? What should be the optimal response of the bank in choosing its default policy on its long-term debt when it is subjected to the potential of a run by depositors and closure by the charter authority? Why should regulators impose bank equity capital standards, and what should be the optimal response of the banks to such standards? We have provided a framework to study these questions in this paper. We believe that these questions are of first order importance to banks, regulators, FDIC, and investors. After the credit crisis of 2008 banks were significantly recapitalized and the regulators have since been grappling with the question of the level and composition of capital that banks should hold.\textsuperscript{17} Basel III has recently announced more stringent capital adequacy standards for banks, and the Swiss National Bank has enforced capital standards that are tougher than Basel III. Admati, et.al (2013) have argued that banks should hold significantly higher equity capital than the levels proposed by Basel III.\textsuperscript{18}

By the nature of the questions that we seek to answer, the literature pertinent to our paper falls under three categories: bank run, optimal capital structure, and bank capital regulation. The bank run literature was pioneered by Diamond and Dybvig (1984) who constructed a formal model in which bank run emerges as one equilibrium, and the FDIC insurance of bank deposits enables the economy to avoid this equilibrium. This literature has subsequently been extended in significant ways by Allen and Gale (1998) and others. In our paper depositors can run in order to make their claims risk-free. There is no panic risk in our full-information model as depositors observe exactly the value of the assets of the bank, and can implement their optimal run (withdrawal) strategy without any risk. In

\textsuperscript{17}Regulators in Europe, for example, are arguing for mandatory bail-in debt. See Coeure (2013), for example.

\textsuperscript{18}In a forthcoming paper, we explore the role of contingent capital in the context of the framework developed in this paper.
this sense we do not have a situation, in which a fraction of the depositors who arrive too late after the onset of a crisis, are left with losses. The challenge of a run in our model, is that the bank must decide how much deposits to hold and how to optimally respond to the ability of the depositors to withdraw any time to make their loans risk-free, in choosing their leverage, debt composition, and optimal default strategy with respect to their long-term creditors. This is similar to the run problem considered by Auh and Sundaresan (2013) in the context of repo financing, but they do not model the FDIC insurance, the charter authority’s closure rules, and capital adequacy standards, which are non-trivial institutional features of any banking environment.

A number of papers have recently contributed to the theory of banks’ optimal capital structure, including the ones by Harding, Liang, and Ross (2009), DeAngelo and Stulz (2013), Allen and Carletti (2013), and Gornall and Strebulaev (2013). Harding, Liang and Ross (2009) consider only deposits, but do not model the issuance of long-term debt. They also assume that the bank does not pay any insurance premium on its deposits. Garnall and Strebulaev (2013) posit that the leverage decisions are jointly made by the banks and its borrowers, and argue that high leverage of banks arises because of low volatility of banks’ assets due to diversification. DeAngelo and Stulz (2013) provide a rationale as to why high leverage may be essential for banks by positing that banks provide liquidity services to financial constrained households and firms that value insurance against liquidity shocks and are prepared to pay a liquidity premium to the banks. We also explore the link between the liquidity services provided by a bank and its value-maximizing endogenous capital and liability structure decisions. Chen, Glasserman and Nouri (2012) focus on contingent capital and bail-in issues in the context of a model of endogenous default where the cash flow process is driven by a mixed diffusion and jump dynamics.

Our paper differs from these contributions in the following ways: first, we allow the equity holders of the bank to arrive at value-maximizing optimal capital and liability structure, which includes both deposits and non-convertible long-term debt: in this sense the composition of deposits and long-term debt as well as the optimal equity capital is endogenously determined. Second, the bank optimally chooses its default strategy on its long-term debt in response to the run risk presented by the presence of depositors. Finally, we determine the optimal equity to asset ratios that banks will choose to hold when it acts to maximize its value, and link this to FDIC’s insurance premium (fair or subsidized) and the charter authorities closure policies. To our knowledge, our paper is the first to examine the optimal response of the bank (in terms of default boundary, equity capital and the composition of liabilities) to the potential for a run by depositors and closure by the charter authority in the presence of FDIC insurance. The FDIC insurance premium depends on the leverage

\footnote{Avdjiev, Katasheva, and Bogdanova (2013) report that during the 2009-2013 period alone, banks have issued $4.1 trillion of unsecured long-term debt. This has always been an important source of funding for banks.}

\footnote{They consider a mix of deposits and long-term debt, but assume that this mix is exogenous.
and liability structure, and the banks’ decisions on leverage and liability structure depend on FDIC premium. We explicitly model this feedback channel. We believe that this is of first order importance in assessing any regulatory policies pertaining to equity capital ratios that banks should be subjected to.

Finally, our paper contributes to the theory of bank regulation. A recent survey of this literature by Santos (2000) motivates regulation as a policy arising out of market failure, in which regulation plays a key role in minimizing run risk, and mitigating banks from taking excessive leverage. We motivate regulation as follows: in an unregulated markets, with full information, value-maximizing banks are shown to take excessive leverage to increase their market values. This leads to a high expected costs of defaults which are dead-weight losses to the society. Provision of FDIC insurance, and closure by charter authority, can serve to lower leverage and thus reduce the expected costs of bank failures. But to bring down further the expected costs of defaults and runs, we argue that bank capital requirements may be necessary.

Our paper is also closely related to the structural framework of Merton (1974, 1977) and Leland (1994). The latter considers unprotected long-term debt and secured debt, which Leland suggests may be interpreted as short-term debt. Leland considers each class of debt separately, and we model the endogenous choice of deposits and long-term debt simultaneously. In addition, we model the deposit insurance provided by the FDIC explicitly with the closure strategy of the charter authority. This allows us to derive the endogenous FDIC insurance premium, taking into account the optimal closure policy and the default boundary. A number of papers, notably Merton (1977), Ronn and Verma (1986) have derived the risk-adjusted FDIC insurance policies, and our paper extends their insights when default and closure policies are endogenous, and the bank optimally responds to FDIC insurance by choosing its liability structure and leverage to maximize its value.

We have not addressed the issue of bank dynamically changing the liability structure as well as its composition of assets. Goldstein, Ju and Leland (2001) have developed a framework to address this important question in the context of a corporate borrower with a single class of debt. An important issue in this context is the ability of banks to de-lever in an orderly fashion in bad states of the world. Extension of their approach to our setting is a much more ambitious task, and one that warrants further research. Subramanian and Yang (2013) consider the question of prudential regulation in a dynamic context, where the bank is able to choose between two projects with different riskiness, but they restrict attention to perpetual debt.

A Appendix

Our model of bank liability structure builds on the framework pioneered by Merton (1974) and extended by Leland (1994) for firms, which issue perpetual debt but do not take de-
posits. We extend their framework to the capital structure of banks, which takes deposits, offer account service, and subject to deposit insurance and regulations.

Following their framework, the value of assets portfolio follows a stochastic process: a geometric Brownian motion in the risk-neutral probability measure:

\[ dV = (r - \delta)V dt + \sigma V dW. \]  

(19)

where \( r \) is the risk-free interest rate, \( \delta \) is the rate of cash flow, \( \sigma \) is the volatility of asset value, and \( W \) is a Wiener process.

For given \( V_b \), consider a security that pays one dollar if and only if \( V \) hits \( V_b \) for the first time. According to Merton (1974), the price of this security, \( P_b \), satisfies the following differential equation

\[ \frac{1}{2} \sigma^2 V^2 P''_b + (r - \delta) V P'_b - r P_b = 0 \]  

(20)

where \( P'_b \) and \( P''_b \) are the first and second partial derivatives of \( P_b \) with respect to asset value \( V \), and boundary conditions \( P_b(V_b) = 1 \) and \( \lim_{V \to \infty} P_b(V) = 0 \). It is well known that the solution of this equation is \( P_b = \frac{V_b}{V} \lambda \), where

\[ \lambda = \sqrt{\frac{(r - \delta - 1)^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{r - \delta - 1}{2}}. \]  

(21)

The solution can be verified directly, observing that \( \lambda \) is the positive root of the following quadratic equation,

\[ \frac{1}{2} \sigma^2 \lambda (1 + \lambda) - (r - \delta) \lambda - r = 0. \]  

(22)

The quadratic equation has two roots in general—one positive and the other negative, but only the positive root is relevant to valuation.

Equity holders of the bank earn the after-tax cash flow, \( \delta V - (1 - \tau)(I + C + C_1) \), until bankruptcy. For given insurance premium \( I \), interests on deposits \( C \), and coupon of subordinated debt \( C_1 \), equity value is a function of asset value, denoted by \( E(V) \). The no-arbitrage pricing equation for equity value before bankruptcy is

\[ \frac{1}{2} \sigma^2 V^2 E'' + (r - \delta) VE' - r E + \delta V - (1 - \tau)(I + C + C_1) = 0, \]  

(23)

where \( E' \) and \( E'' \) are the first and second partial derivatives of \( E \) with respect to asset value \( V \). In this equation, we assume \( I \geq 0 \) in general, but if setting \( I = 0 \) gives the pricing equation of equity value in the case of unregulated banks without FDIC insurance.

If asset value is infinitely large, the cash flows are risk-free and thus equity value should become approximately the difference between asset value and after-tax value of cash outflows: \( V - (1 - \tau)(I + C + C_1)/r \). If asset value drops to endogenous default boundary \( V_d \),
equity value satisfies the following two conditions: \( E(V_d) = 0 \) and \( E'(V_d) = 0 \), according to Leland (1994). Recall that bankruptcy boundary is \( V_b = \max\{V_d, V_a\} \), where \( V_a = \kappa D \) is the boundary of bank run or bank closure. When bank is closed, equity is wiped out and thus \( E(V_a) = 0 \). Therefore, the boundary conditions of equity value are

\[
\lim_{V \to \infty} E(V) - [V - (1 - \tau)(I + C + C_1)/r] = 0 \tag{24}
\]

\[
E(V_b) = 0 \tag{25}
\]

\[
E'(V_b) = 0 \quad \text{if} \quad V_d > V_a. \tag{26}
\]

The value of subordinated debt \( D_1 \) is also a function of asset value \( V \). Let \( D_1' \) and \( D_1'' \) be the first and second partial derivatives of \( D_1 \) with respect to \( V \). The no-arbitrage pricing restriction to the value of the debt \( D_1 \) is

\[
\frac{1}{2} \sigma^2 V^2 D_1'' + (r - \delta)V D_1' - rD_1 + C_1 = 0. \tag{27}
\]

If asset value is infinitely large, the subordinated debt becomes risk-free, and thus the debt value approaches \( C_1/r \). Recall that at bankruptcy, payoff to debt holders is \( [(1 - \alpha)V_b - D]^+ \). Therefore, the boundary conditions of subordinated debt value are

\[
\lim_{V \to \infty} D_1(V) = C_1/r \tag{28}
\]

\[
D_1(V_b) = [(1 - \alpha)V_b - D]^+. \tag{29}
\]

The solutions to above equations and boundary conditions are presented in Theorem 1. Derivation of the solutions is similar to those in Leland (1994) as detailed in Section A.1. To simplify the derivation of optimal liability structure, we introduce the following notations:

\[
x = C_1/C, \quad c = C/(rV), \tag{30}
\]

\[
v_a = rV_a/C, \quad v_d = rV_d/C, \quad v_b = rV_b/C, \tag{31}
\]

\[
\tau = \eta/(r - \eta), \quad \theta = (1 - \tau)\lambda/(1 + \lambda). \tag{32}
\]

We refer to \( x \) as the liability ratio of a given liability structure and \( c \) as the deposit liability scaled by asset. With these notations, the state price of bankruptcy is simply \( P_b = (v_b c)^\lambda \). Then, by Theorem 1, the rescaled boundaries are

\[
v_a = \kappa (1 + \iota) \tag{33}
\]

\[
v_d = \theta (1 + i + x) \tag{34}
\]

\[
v_b = \max\{\kappa (1 + \iota), \theta (1 + i + x)\} \tag{35}
\]

Furthermore, equation (8) can be written as a function of \( c \)

\[
i = \omega [1 - (1 - \beta)\kappa]^+ (1 + \iota) (v_a c)^\lambda /[1 - (v_a c)^\lambda]. \tag{36}
\]

34
We can also express deposits, subordinated debt, and bank values in Theorem 1 as ratios to asset value:

\[
\frac{D}{V} = (1+i)c \tag{37}
\]

\[
\frac{D_1}{V} = \{x[1-(vb^c)\lambda] + [(1-\phi)vb - (1+i)]+(vb^c)\lambda\}c \tag{38}
\]

\[
\frac{F}{V} = 1 + \left[1-(vb^c)\lambda\right] \left[(i + \tau(1+x) - (1-\tau)i)c - (vb^c)\lambda \min\{\phi vb, vb - (1+i)\}c \right] = f(x,c) \tag{39}
\]

The last equation is the bank value scaled by asset and we denote it by \(f(x,c)\). Choosing \((C, C_1)\) to maximize \(F\) is equivalent to choosing the duplet \((x, c)\) to maximize \(f\). Once we obtain the optimal \((x^*, c^*)\), the optimal \((C^*, C_1^*)\) can be obtained easily from \(C^* = c^*rV\) and \(C_1^* = x^*C^*\). Also notice that \(V_a < V_d\) if and only if \(v_a < v_d\).

For the case of unregulated bank without deposit insurance, we should setting \(i = 0\) and \(\phi = \alpha\) in equations (33)–(39). Rational bank run implies we also have \(\kappa = 1/(1-\alpha)\).

**Lemma 1** Assume \(i = 0\), \(\phi = \alpha\), and \(\kappa = 1/(1-\alpha)\). The scaled bank value \(f(x,c)\) has a unique maximum \((x^*, c^*)\). At the maximum, the liability ratio is

\[
x^* = r(1+\lambda)/[\lambda(r-\eta)(1-\alpha)(1-\tau)] - 1, \tag{40}
\]

which implies \(v_a^* = v_d^*\), the state price is

\[
\pi = \frac{1}{1 + \lambda} \cdot \frac{\eta\theta (1-\alpha)+r\tau}{\eta\theta (1-\alpha)+r(\tau+\alpha\theta)}, \tag{41}
\]

and the scaled deposit liability is \(c^* = \pi^{1/\lambda}(1-\alpha)(r-\eta)/r\).

The proof of the lemma is presented in Appendix A.3. From this lemma, the optimal capital structure in an unregulated bank without deposit insurance can be solved analytically. Theorem 3 presents the economic characterization of the optimal liability structure.

Now, consider banks with deposit insurance and under FDIC regulation. We maximize the scaled bank value \(f\) in equation (39) by choosing \((x, c)\). In this case, \(i\) endogenously depends on \(c\) through equation (36).

**Lemma 2** Assume \(\kappa \geq 1\) and that \(i\) is a function of \(c\) as defined in equation (36). The liability structure \((x^*, c^*)\) maximizes \(f(x,c)\) if and only if \(v_a^* \geq v_d^*\). In the optimal, the state price of bankruptcy \(\pi\) satisfies the following quartic equation

\[
\varphi_0 + \varphi_1 \pi + \varphi_2 \pi^2 + \varphi_3 \pi^3 + \varphi_4 \pi^4 = 0 \tag{42}
\]

with each \(\varphi_j\) being a simple algebraic function of the following exogenous parameters: \(r\), \(\sigma\), \(\delta\), \(\tau\), \(\eta\), \(\beta\), \(\kappa\), and \(\omega\), and optimal liability ratio and scaled deposit liability are

\[
x^* = \frac{r}{r-\eta} \left\{\frac{\kappa(1+\lambda)}{(1-\tau)\lambda} - \omega[1-(1-\beta)\kappa]^{+} \frac{\pi}{1-\pi}\right\} - 1 \tag{43}
\]

\[
c^* = \pi^{1/\lambda}(r-\eta)/(r\kappa) \tag{44}
\]
Appendix A.4 presents the mathematical proof. This lemma prepares the mathematical facts for Theorem 4, whereas the theorem focuses on characterizing the economic features of the optimal liability structure for banks paying endogenous deposit insurance premium and being regulated by FDIC and its charter authority.

Equity requirement imposes a cap on leverage ratio. The leverage ratio is \( l = (D + D_1)/V \). It follows from equations (37) and (38) that, given other parameters, the leverage ratio \( l \) is a function of \( (x, c) \):

\[
l(x, c) = c\{1 + \iota + x[1 - (v_b c)^\lambda] + [(1 - \phi) v_b - (1 + \iota)]^+(v_b c)^\lambda\}. \tag{45}
\]

Then, the equity requirement is \( l(x, c) \leq 1 - \zeta \). The optimal liability structure subject to equity requirement is

\[
(x^\dagger, c^\dagger) = \text{argmax}\{f(x, c) : l(x, c) \leq 1 - \zeta\}. \tag{46}
\]

If the optimal structure \((x^\dagger, c^\dagger)\) subject to equity requirement is not the same as the unrestricted optimal structure \((x^*, c^*)\), then the constraint of equity requirement must be binding. In this case,

\[
(x^\dagger, c^\dagger) = \text{argmax}\{f(x, c) : l(x, c) = 1 - \zeta\}. \tag{47}
\]

**Lemma 3** Assume \( i = 0 \), \( \phi = \alpha \), and \( \kappa = 1/(1 - \alpha) \). The scaled bank value \( f(x, c) \) has a unique maximum \((x^\dagger, c^\dagger)\) subject to equity requirement. If the equity requirement is binding at the maximum, then \( v_d^\dagger \leq v_a^\dagger \). In the case of \( v_d^\dagger < v_a^\dagger \), the state price of bankruptcy is

\[
\pi = \frac{1}{1 + \lambda} \cdot \frac{\eta(1 - \tau)(1 - \alpha)}{r + \tau(1 - \alpha)} , \tag{48}
\]

but in the case of \( v_d^\dagger = v_a \), the state price of bankruptcy is \( \pi = z^\lambda \), where \( z \) solves the algebraic equation

\[
\left[1 - \frac{r(1 + \lambda)}{(r - \eta)(1 - \alpha)(1 - \tau)\lambda}\right]z^{1+\lambda} + \left[\iota + \frac{r(1 + \lambda)}{(r - \eta)(1 - \alpha)(1 - \tau)\lambda}\right]z
- \frac{(1 - \zeta)r}{(r - \eta)(1 - \alpha)} = 0. \tag{49}
\]

The optimal liability structure is

\[
x^\dagger = \frac{r}{(r - \eta)(1 - \pi)}\left[\frac{1 - \zeta}{(1 - \alpha)\pi^{1/\lambda}} - 1\right] \quad \text{if} \quad v_d^\dagger < v_a^\dagger \tag{50}
\]

\[
x^\dagger = \frac{r(1 + \lambda)}{(r - \eta)(1 - \alpha)(1 - \tau)\lambda} \quad \text{if} \quad v_d^\dagger = v_a^\dagger \tag{51}
\]

\[
c^\dagger = \pi^{1/\lambda}(1 - \alpha)(r - \eta)/r . \tag{52}
\]

Appendix A.5 contains the proof of this lemma.
A.1 Proof of Theorem 1

The general form of solution to pricing equation (23) is
\[ E = \delta V - (1 - \tau)(I + C + C_1)/r + a_1 V + a_2 V^{-\lambda} \]  \hspace{1cm} (53)
where \( \lambda \) is a solution to equation (22), and \( a_1 \) and \( a_2 \) can be any two constants. The two constants and the default boundary together are determined by the three boundary conditions for equity value. Boundary condition (24) immediately implies \( \lambda > 0 \) and \( a_1 = 1 - \delta \). It then follows from boundary condition (26) that \( 1 - \lambda a_2 V_b^{-(\lambda+1)} = 0 \), which gives \( a_2 = V_b^{1+\lambda}/\lambda \). Further more, the boundary condition (25) implies \( V_b - (1 - \tau)(C + C_1)/r + a_2 V_b^{-\lambda} = 0 \). Substituting out \( a_2 \) in the preceding equation and solving for \( V_b \), we obtain equation (1). Substituting \( a_1 = 1 - \delta \) and \( a_2 = V_b^{1+\lambda}/\lambda \) into equation (53), we arrive at equation (2).

The general form of solution to pricing equation (27) is
\[ D = C_1/r + b_1 V + b_2 V^{-\lambda}, \]  \hspace{1cm} (54)
where \( \lambda \) is a solution to equation (22), and \( b_1 \) and \( b_2 \) can be any two constants. Given bankruptcy boundary \( V_b \), the constants, \( b_1 \) and \( b_2 \), are determined by boundary conditions of subordinated debt. Boundary condition (28) implies \( \lambda > 0 \) and \( b_1 = 0 \). Boundary condition (29) then implies \( C_1/r + b_2 V_b^{-\lambda} = [(1 - \alpha)V_b - D]^+ \), which gives
\[ b_2 = \{[(1 - \alpha)V_b - D]^+ - C_1/r\} V_b^\lambda. \]  \hspace{1cm} (55)
Substituting equation (55) and \( b_1 = 0 \) into equation (54), we obtain equation (3).

Bank value is \( F = D + D_1 + E \). Substituting equations (3) and (2), we obtain
\[ F = V + (1 - P_b)[\tau C_1 - (1 - \tau)(I + C)]/r + D + P_b[(1 - \phi)V_b - D]^+ - P_b V_b. \]  \hspace{1cm} (56)
Using \( D = C/(r - \eta) \), we can express deposits as \( D = (1 - P_b)[r/(r - \eta)]C/r + P_b D \), which is used to rewrite equation (56) into equation (4).

A.2 Proof of Theorem 2

Let \( Q \) be the value of the insurance product to banks. The pricing equation of the insurance product is
\[ \frac{1}{2} \sigma^2 V^2 Q'' + (r - \delta) V Q' - rQ - I = 0, \]  \hspace{1cm} (57)
where \( Q' \) and \( Q'' \) represents the first and second partial derivatives of \( Q \) with respect to \( V \). The general solution to the differential equation is \( Q(V) = -I/r + a_1 V + a_2 V^{-\lambda} \), where \( a_1 \) and \( a_2 \) can be any constants.
The boundary conditions of the value of the insurance product are \( \lim_{V \to \infty} Q = -I/r \) and \( Q(V_a) = [D - (1 - \beta)V_a]^+ \), where \( V_a = \kappa D \). The upper boundary condition restricts \( a_1 \) to zero, and the lower boundary condition implies \(-I/r + a_2 V_a^{-\lambda} = [D - (1 - \beta)V_a]^+ \). Solving \( a_2 \) from the lower boundary condition and substituting it and \( a_1 = 0 \) back into the formula of \( Q(V) \), we obtain

\[
Q(V) = -(1 - P_a)I/r + [D - (1 - \beta)V_a]^+ P_a ,
\]

(58)

where \( P_a = [V_a/V]^\lambda \).

Since fair insurance premium should make the insurance product worth zero, we should have \( Q(V) = 0 \). It follows that the fair insurance premium \( I \) must satisfy

\[
(1 - P_a)I = r[D - (1 - \beta)V_a]^+ P_a .
\]

(59)

Finally, we obtain equation (5) by substituting \( V_a = \kappa D \) and then factoring \( D \) out from the truncation function.

### A.3 Proof of Lemma 1

It follows from \( i = 0, \phi = \alpha, \kappa = 1/(1 - \alpha) \), and equation (35) that \( v_b = \max\{(1 + \nu)/(1 - \alpha), \theta(1 + x)\} \). This further implies \( \phi v_b \leq v_b - (1 + \nu) \), which implies equation (39) to

\[
f = 1 + \left\{ \nu + \tau(1 + x) - (v_b c)\lambda\left[\nu + \tau(1 + x) + \alpha v_b\right] \right\} c
\]

(60)

It follows that the first-order condition for \( c \) to be optimal is

\[
\nu + \tau(1 + x) - (1 + \lambda)\left[\nu + \tau(1 + x) + \alpha v_b\right](v_b c)\lambda = 0 .
\]

(61)

Notice that \( v_b = \theta(1 + x) \) if and only if \( x \geq x^* \), where \( x^* = (1 + \nu)/[\theta(1 - \alpha)] - 1 \). Otherwise, \( v_b = (1 + \nu)/(1 - \alpha) \). If \( x < x^* \), then the partial derivative of \( f \) respect to \( x \) is

\[
f'_x = \tau[1 - (b_v c)\lambda]c > 0
\]

(62)

which implies that \( f \) is an increasing function in \( x \) for \( x < x^* \). If \( x > x^* \), then the partial derivative of \( f \) with respect to \( x \) is

\[
f'_x = \left\{ \tau - [\nu/(1 + x) + (1 + \lambda)(\nu + \alpha \theta)](v_b c)\lambda \right\} c .
\]

(63)

In the case of \( x > x^* \), condition (61) becomes

\[
\nu + \tau(1 + x) - (1 + \lambda)\left[\nu + (\tau + \alpha \theta)(1 + x)\right](v_b c)\lambda = 0 .
\]

(64)

Imposing this condition equation (63) gives

\[
\left[ f'_x \right]_{f'_x = 0} = -\frac{\tau}{1 + x}[1 - (v_b c)\lambda]c < 0 .
\]

(65)
Thus, \( f \) is a decreasing function in \( x \) for \( x > x^* \) if we always keep \( c \) optimal relative to \( x \).

Therefore, \( x^* \) is the optimal point for \( x \). At this point, \( v_a^* = v_d^* = v_b^* = (1 + \iota)/(1 - \alpha) \). It follows that \( P_a^* = P_d^* = P_b^* \). We can solve \( P_b^* \) from equation (61) as

\[
P_b^* = \frac{1}{1 + \lambda} \cdot \frac{\iota + \tau (1 + x^*)}{\iota + \tau (1 + x^*) + \alpha v_b^*}.
\]

After substituting out \( x^* \) and \( v_b^* \), the above formula gives

\[
P_b^* = \frac{1}{1 + \lambda} \cdot \frac{\iota \theta (1 - \alpha) + \tau (1 + \iota)}{\iota \theta (1 - \alpha) + (\tau + \alpha \theta)(1 + \iota)}.
\]

Thus, we define the right-hand size of the above equation as \( \pi \). From \((v_b^* c^*)^\lambda = \pi\), we obtain \( c^* = \pi^{1/\lambda}(1 - \alpha)/(1 + \iota) \). After substituting out \( \iota \) and \( \theta \) using equation (32), we arrive at the formulas of \( x^* \) and \( c^* \) in the lemma, which completes the proof.

### A.4 Proof of Lemma 2

We first show that \( v_a > v_d \) cannot be optimal. Observe that \( v_a > v_d \) if and only if \( \theta(1 + i + x) < (1 + \iota)\kappa \). We then have \( v_b = v_a = (1 + \iota)\kappa \) in this case. Since \( v_b \) is independent of \( x \) in this case, the partial derivative of \( f \) with respect to \( x \) is

\[
f'_x = \tau [1 - (v_b c)^\lambda] c.
\]

Because the state price \((v_b c)^\lambda\) is smaller than 1 when the bank is not closed, we have \( f'_x > 0 \). This implies that bank value can increase if we raise the liability ratio \( x \). Thus, such \( x \) cannot be optimal.

It follows from equation (36) that the derivative of \( i \) with respect to \( c \) is

\[
i'_c = \lambda ic^{-1}/[1 - (v_a c)^\lambda].
\]

If \( \kappa < 1/(1 - \beta) \), both \( i \) and \( i'_c \) are positive. Moreover, both \( i \) and \( i'_c \) converge to zero when \( \kappa \) rises to \( 1/(1 - \beta) \) while other parameters are fixed. If \( \kappa \geq 1/(1 - \beta) \), both \( i \) and \( i'_c \) are zero. Also notice that \( i \) and \( i'_c \) converge to zero when \( \omega \) goes down to zero while \( \kappa \) and other parameters are fixed.

Since an optimal liability structure must satisfy \( v_d \geq v_a \), let us consider \( x \) that satisfies \( \theta(1 + i + x) \geq (1 + \iota)\kappa \). In this case \( v_b = \theta(1 + i + x) \). Now suppose the strict inequality \( \theta(1 + i + x) > (1 + \iota)\kappa \) holds. It follows that \( v_d > v_a \) and \( \phi = \alpha \). Then, we have

\[
\min \{ \phi v_b, v_b - (1 + \iota) \} = \begin{cases} 
\quad v_b - (1 + \iota) & \text{if } v_b \leq (1 + \iota)/(1 - \alpha) \\
\quad \alpha v_b & \text{if } v_b > (1 + \iota)/(1 - \alpha).
\end{cases}
\]

Notice that \((1 + \iota)\kappa < (1 + \iota)/(1 - \alpha)\) if \( \kappa < 1/(1 - \beta) \) and \( \beta \leq \alpha \).
For the case of \( v_b \leq (1 + \iota)/(1 - \alpha) \), we use the first part of equation (70) and substitute \( v_b = \theta(1 + i + x) \) into equation (39) to obtain

\[
 f = 1 + c\{\iota - i + \tau(1 + i + x) + \left[1 + i - (\tau + \theta)(1 + i + x)\right](v_b c)^\lambda\}. \quad (71)
\]

Its partial derivative with respect to \( x \) is, using the definition of \( \theta \),

\[
 f'_x = c\left\{\tau - \frac{\lambda x}{(1 + i + x)}\right\}(v_b c)^\lambda. \quad (72)
\]

In view of equation (69), the partial derivative of \( f \) with respect to \( c \) is

\[
 f'_c = 1 + \iota - \left[1 + i - \tau(1 + i + x)\right][1 - (v_b c)^\lambda] - \lambda x (v_b c)^\lambda - \left\{\left(1 - \tau\right)[1 - (v_b c)^\lambda] + \left[\frac{\lambda x}{(1 + i + x)}\right](v_b c)^\lambda\right\}\lambda i/[1 - (v_\alpha c)^\lambda]. \quad (73)
\]

Let \( c_x \) be the optimal \( c \) for given \( x \), then \( f'_c(x, c_x) = 0 \) in equation (73), from which we can solve \((v_b c_x)^\lambda\) as a function of \( i \) and \( x \). Substituting this function into equation (72), we can show \( f'_x(x, c_x) < 0 \) for \( \kappa \geq 1 \). Therefore, in the case of \( v_\alpha < v_d < (1 + \iota)/(1 - \alpha) \), lowering \( x \) will increase \( f \), and thus such \( x \) is not optimal.

For the case of \( v_b > (1 + \iota)/(1 - \alpha) \), substitution of \( v_b = \theta(1 + i + x) \) and the second part of equation (70) into equation (39) gives

\[
 f = 1 + c\left\{\iota - i + \tau(1 + i + x) - \left[\iota - i + (\tau + \alpha \theta)(1 + i + x)\right](v_b c)^\lambda\right\}. \quad (74)
\]

The partial derivative of \( f \) with respect to \( x \) is

\[
 f'_x = c\left\{\tau - \frac{(1 + \lambda)(\tau + \alpha \theta) + \lambda(\iota - i)/(1 + i + x)}{(1 + i + x)}\right\}(v_b c)^\lambda. \quad (75)
\]

The partial derivative of \( f \) with respect to \( c \) is

\[
 f'_c = (\iota - i)[1 - (v_b c)^\lambda] + (1 + i + x)\left\{\tau - \left[(1 + \lambda)(\tau + \alpha(1 - \tau) + \lambda(\iota - i)/(1 + i + x)\right](v_b c)^\lambda\right\}
\]

\[
 - \left\{(1 - \tau)[1 - (v_b c)^\lambda] + \lambda\left[\frac{\tau + \alpha(1 - \tau) + (\iota - i)/(1 + i + x)}{(1 + i + x)}\right](v_b c)^\lambda\right\}
\]

\[
 \cdot \lambda i/[1 - (v_\alpha c)^\lambda]. \quad (76)
\]

Let \( c_x \) be the optimal \( c \) relative to \( x \), we have \( f'_c = 0 \). Then, setting \( f'_c = 0 \) in equation (76), we can solve \((v_b c_x)^\lambda\) as a function of \( i \) and \( x \). Substituting this function into equation (75), we can show \( f'_x(x, c_x) < 0 \) for \( \kappa \geq 1 \), which implies \( v_b > (1 + \iota)/(1 - \alpha) \) cannot be optimal if \( \kappa \geq 1 \).

The above two cases show that if \( \kappa \geq 1 \), we have \( f'_x < 0 \) for all \( x \) that satisfy \( \theta(1 + i + x) > (1 + \iota)\kappa \), if \( c \) is kept to be optimal relative to \( x \). Therefore, \( \theta(1 + i + x) > (1 + \iota)\kappa \) cannot be
optimal because reducing \( x \) will add value to the bank. Consequently, the optimal choice for \( x \) must satisfy \( \theta(1 + i + x) = (1 + i)\kappa \), which implies \( v_d = (1 + i)\kappa \) and thus \( v_a = v_d \).

Since the optimality condition requires \( v_a = v_d \), the default and closure boundaries have the same probability to hit: \( \pi = (v_a c)^\lambda = (v_b c)^\lambda \). This means \( v_a = \pi^{1/\lambda}/c \). Then, in view of equation (33), we have \((1 + i)\kappa = \pi^{1/\lambda}/c \). The above gives equation (44). Equations (36) and (33) imply

\[
i = (1 + i)\omega[1 - (1 - \beta)\kappa]^+\pi/(1 - \pi)
\]  
which gives equation (43). It follows from equation (73) that

\[
f'_c = 1 + \iota - [1 + i - \tau(1 + i)\kappa/\theta][1 - (1 + \lambda)\pi] - (1 - \tau)\lambda i + \left[\theta(1 + i)/(1 + \iota)\kappa - 1\right] \lambda^2 i \cdot \frac{\pi}{1 - \pi}
\]  
(78)

Substituting equation (77) and multiplying by \((1 - \pi)^2\), we obtain a polynomial of four degrees in \( \pi \), where the five coefficients of the polynomial are simple algebraic functions of \( \lambda, \iota, \kappa, \tau, \theta \) and \( \omega \). This completes the proof.

### A.5 Proof of Lemma 3

With \( i = 0, \phi = \alpha, \) and \( \kappa = 1/(1 - \alpha) \), the scaled bank value is

\[
f(x, c) = 1 + [1 - (v_b c)^\lambda][(\iota + \tau(1 + x)]c - (v_b c)^\lambda \min\{\alpha v_b, v_b - (1 + i)\} c.
\]  
(79)

First, consider the case of \( \theta(1 + x) > v_a \). In this case, \( v_b = \theta(1 + x) \). Equation (79) becomes

\[
f(x, c) = 1 + c\left[\tau + (1 + \lambda)[v_b c]^{\lambda}\right] - \alpha\theta(1 + x)(v_b c)^{\lambda/\kappa} - (v_b c)^{\lambda} - \alpha\theta(1 + x)(v_b c)^{\lambda}. \]

(80)

The partial derivatives of \( f(x, c) \) are

\[
f'_c(x, c) = c\left[\tau - [\lambda\iota/(1 + x)] + (1 + \lambda)(\tau + \alpha\theta)(v_b c)^{\lambda}\right]
\]  
(81)

\[
f'_c(x, c) = \iota + \tau(1 + x) - (1 + \lambda)[\iota + (\tau + \alpha\theta)(1 + x)](v_b c)^{\lambda}.
\]  
(82)

Let \( c_1(x) \) be value of \( c \) such that \( f'_c = 0 \) for given \( x \). Then, \( f'_c(x, c) > 0 \) for \( c < c_1(x) \) and \( f'_c(x, c) < 0 \) for \( c > c_1(x) \).

\[
[v_b c_1(x)]^{\lambda} = \frac{\iota + \tau(1 + x)}{1 + \lambda \iota + (\tau + \alpha\theta)(1 + x)}.
\]  
(83)

Let \( x^\star \) be the value of \( x \) such that \( \theta(1 + x) = (1 + i)/(1 - \alpha) \). By Lemma 1, \((x^\star, c_1(x^\star))\) maximizes \( f \).
The formula of leverage ratio becomes
\[ l(x, c) = c \{ [1 - (v_b c)^\lambda] + (1 + x)[1 - (1 - (1 - \alpha)\theta)(v_b c)^\lambda] \}. \] (84)

Let \( c_2(x) \) be the value of \( c \) such that \( l(x, c) = 1 - \zeta \) for given \( x \). Then,
\[ [v_b c_2(x)]^\lambda = \frac{\tau + 1 + x - (1 - \zeta)/c}{\tau + (1 - (1 - \alpha)\theta)(1 + x)}. \] (85)

The partial derivatives of \( l(x, c) \) are
\[ l'_x(x, c) = c \{ 1 - [\lambda\tau/(1 + x) + (1 + \lambda)(1 - (1 - \alpha)\theta)](v_b c)^\lambda \} \] (86)
\[ l'_c(x, c) = \tau + 1 + x - (1 + \lambda)[\tau + (1 - (1 - \alpha)\theta)(1 + x)](v_b c)^\lambda. \] (87)

If \( l(x, c_1(x)) < 1 - \zeta \), let \( g_1(x) = f(x, c_1(x)) \). It follows that
\[ g'_1(x) = f'_x(x, c_1(x)) + f'_c(x, c_1(x))c'_1(x) = -c\tau/(1 + x)[1 - (v_b c)^\lambda] < 0. \] (88)
and thus reducing \( x \) increases \( f \), which implies \( x \) cannot be optimal. If \( l(x, c_1(x)) \geq 1 - \zeta \). For any \( c < c_1(x) \), by equation (87) we have
\[ l'_c(x, c) > l'_c(x, c_1(x)) = \{ 1 - [1 - (1 - \alpha)\theta]\}(1 + x) > 0. \] (89)
Thus, \( l(x, c) \leq 1 - \zeta \) implies \( c < c_1(x) \), which further implies \( f'_x(x, c) > 0 \). This means \( c_2(x) \) maximizes \( f(x, c) \) subject to \( l(x, c) \leq 1 - \zeta \). Now, let \( g_2(x) = f(x, c_2(x)) \). Then,
\[ g'_2(x) = l'_x(x, c_2(x)) + l'_c(x, c_2(x))c'_2(x). \] (90)

Differentiating the constraint (84) with respect to \( x \), we obtain
\[ 0 = c'_2(x) \{ [\tau + (1 + x)][1 - (1 + \lambda)(v_b c_2(x))^\lambda] + (1 - \alpha)\theta(1 + x)(1 + \lambda)(v_b c)^\lambda \}
+ c_2(x) \{ 1 - (1 + \lambda)(v_b c_2(x))^\lambda + (1 - \alpha)\theta(1 + \lambda)(v_b c_2(x))^\lambda
- [\lambda\tau/(1 + x)](v_b c_2(x))^\lambda \}. \] (91)

The above is equivalent to
\[ [(1 + x)c'_2(x) + c_2(x)] \{ [\tau + (1 + x)][1 - (1 + \lambda)(v_b c_2(x))^\lambda]
+ (1 - \alpha)\theta(1 + \lambda)(v_b c)^\lambda \} = c'_2(x)[\tau/(1 + x)][1 - (v_b c_2(x))^2]. \] (92)

Subtracting equation (91) from equation (90), we obtain
\[ g'_2(x) = -[c + (1 + x)c'_1]\{ (1 - \tau)[1 - (1 + \lambda)(v_b c)^\lambda] + \theta(1 + \lambda)(v_b c)^\lambda \}. \] (93)
Substituting the definition of $\theta$, we simplify the above into
\[ g'_2(x) = -[c + (1 + x)c'](1 - \tau)[1 - (1 + \lambda)(v_b c)^\lambda]. \] (94)

Notice that equation (92) implies $c_2(x) + (1 + x)c'_2(x) > 0$. It then follows that $g'_2(x) < 0$. Therefore, decreasing $x$ while keep the capital requirement binding increases $f$, and thus $x$ cannot be optimal.

Now, consider the case of $\theta(1 + x) \leq v_a$. In this case, $v_b = v_a$. It follows that $[(1 - \alpha)v_b - (1 + \iota)]^+ = 0$ and simplifies equation (79) to
\[ f(x, c) = 1 + c \left\{ [\iota + \tau(1 + x)] [1 - (v_a c)^\lambda] - (\alpha v_a)(v_a c)^\lambda \right\}. \] (95)

The derivatives of $f$ are
\[ f'_2(x, c) = c\tau[1 - (v_a c)^\lambda] \] (96)
\[ f'_1(x, c) = [\iota + \tau(1 + x)] [1 - (1 + \lambda)(v_a c)^\lambda] - (\alpha v_a)\lambda(v_a c)^\lambda \] (97)

In this case, the formula of leverage ratio (45) is simplified to
\[ l(x, c) = c \left\{ 1 + \iota + x[1 - (v_a c)^\lambda] \right\}. \] (98)

The derivatives of $l(x, c)$ are
\[ l'_2(x, c) = c[1 - (v_a c)^\lambda] > 0 \] (99)
\[ l'_1(x, c) = 1 + \iota + x[1 - (1 + \lambda)(v_a c)^\lambda]. \] (100)

If $\theta(1 + x) < v_a$ and $(x, c)$ is optimal in this case, the capital requirement must be binding because $l_x(x, c) > 0$ and otherwise we can increase $f$ by increasing $x$. Let $c_3(x)$ be the value of $c$ such that $l(x, c_3(x)) = 1 - \zeta$ for give $x$ with $\theta(1 + x) < v_a$. Differentiate the binding constraint with respect to $x$, we obtain
\[ c_3(x)[1 - (v_a c_3(x))^\lambda] + c'_3(x) \left\{ 1 + \iota + [1 - (1 + \lambda)(v_a c_3(x))^\lambda] \right\} = 0. \] (101)

Let $g_3(x) = f(x, c_3(x))$. We have
\[ g'_3(x) = f'_2(x, c_3(x)) + f'_1(x, c_3(x))c'_3(x) \]
\[ = c'_3(x) \left\{ (1 - \tau)\iota - [\iota + v_a](1 + \lambda)(v_a c_3(x))^\lambda \right\}. \] (102)

The first-order condition $g'_3(x) = 0$ implies $(v_b c)^\lambda = \pi$, where $\pi$ is given in equation (48). It follows that $c_3(x) = \pi^{1/\lambda}(1 - \alpha)/(1 + \iota)$, which leads to equation (52) in view of equation (32). Substituting this $c$ into constraint (98), we solve for $x^\dagger$ and obtain equation (50). Function $g_3(x)$ achieves maximum at $x^\dagger$ because it is in fact increasing for $x < x^\dagger$ and decreasing for $x > x^\dagger$. The reason is as follows. Equation (101) actually indicates that
c_3'(x) < 0, i.e., c_3(x) is a decreasing function of x. Then, (v_0c_3(x))^\lambda > \pi for x < x^\dagger and (v_0c_3(x))^\lambda < \pi for x > x^\dagger. In view of equation (102) and c_3(x)' < 0, we know g_3'(x) > 0 for x < x^\dagger and g_3'(x) < 0 for x > x^\dagger.

If g_3'(x) > 0 for all x such that \theta(1+x) < v_a, the optimal x^\dagger must satisfy \theta(1+x^\dagger) = v_a, which gives equation (51). Multiplying v_a through l(x^\dagger,c_3(x^\dagger)) = 1 - \zeta and letting z = v_0c_3(x^\dagger), we obtain equation (49). Finally, v_0c_3(x^\dagger) = \pi gives equation (52) and completes the proof.

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Table 1: Exogenous Parameters Variables

The exogenous variables in the model are pre-specified, not determined by either valuation or optimization in the model. The third column of the table lists the range allowed theoretically in the model. The last column lists the values assumed for the baseline case.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Range</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset volatility</td>
<td>$\sigma$</td>
<td>$(0, \infty)$</td>
<td>10.00%</td>
</tr>
<tr>
<td>Asset cash flow</td>
<td>$\delta$</td>
<td>$[0, \infty)$</td>
<td>3.00%</td>
</tr>
<tr>
<td>Asset value</td>
<td>$V$</td>
<td>$(0, \infty)$</td>
<td>1.00 $\text{billion}$</td>
</tr>
<tr>
<td>Bank service income</td>
<td>$\eta$</td>
<td>$(0, r)$</td>
<td>0.50%</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>$(0, \infty)$</td>
<td>3.00%</td>
</tr>
<tr>
<td>Corporate tax rate</td>
<td>$\tau$</td>
<td>$(0, 1)$</td>
<td>30.00%</td>
</tr>
<tr>
<td>Court bankruptcy cost</td>
<td>$\alpha$</td>
<td>$(0, 1)$</td>
<td>25.00%</td>
</tr>
<tr>
<td>FDIC liquidation cost</td>
<td>$\beta$</td>
<td>$(0, 1)$</td>
<td>25.00%</td>
</tr>
<tr>
<td>Bank closure rule</td>
<td>$\kappa$</td>
<td>$[1, \infty)$</td>
<td>102.00%</td>
</tr>
<tr>
<td>Insurance subsidy</td>
<td>$\omega$</td>
<td>$[0, 1]$</td>
<td>100.00%</td>
</tr>
<tr>
<td>Equity requirement</td>
<td>$\zeta$</td>
<td>$(0, 1)$</td>
<td>7.00%</td>
</tr>
</tbody>
</table>

Table 2: Endogenous Variables in Optimal Structure.

Numerical values of endogenous variables in optimal liability structures are calculated for unregulated banks and FDIC-insured banks, given the exogenous parameters in Table 1. The banks are not subject to equity requirement. The first two columns list the definitions of endogenous variables. The third column reports the optimal values of endogenous variables for an unregulated bank. The next column reports the optimal values for an FDIC-insured bank. The last column presents the difference between the optimal values for the two types of banks. All numerical values are presented in percentage points.

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Definition</th>
<th>Unregulated</th>
<th>FDIC-insured</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charter value</td>
<td>$(F - V)/V$</td>
<td>25.67</td>
<td>27.10</td>
<td>1.43</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>$D/V$</td>
<td>37.61</td>
<td>48.43</td>
<td>10.82</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>$D_1/V$</td>
<td>56.97</td>
<td>46.74</td>
<td>-10.24</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>$E/V$</td>
<td>31.09</td>
<td>31.93</td>
<td>0.84</td>
</tr>
<tr>
<td>Leverage retio</td>
<td>$(D + D_1)/V$</td>
<td>94.58</td>
<td>95.17</td>
<td>0.58</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>$V_a/V$</td>
<td>50.15</td>
<td>49.40</td>
<td>-0.75</td>
</tr>
<tr>
<td>Default boundary</td>
<td>$V_d/V$</td>
<td>50.15</td>
<td>49.40</td>
<td>-0.75</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>$P_\phi V_b/V$</td>
<td>3.15</td>
<td>3.01</td>
<td>-0.14</td>
</tr>
<tr>
<td>Credit spread</td>
<td>$s$</td>
<td>1.01</td>
<td>0.97</td>
<td>-0.04</td>
</tr>
<tr>
<td>Insurance premium</td>
<td>$I/V$</td>
<td>0.00</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Table 3: Optimal Structure of Unregulated Banks

Numerical values of endogenous variables in the optimal liability structures of unregulated banks are calculated for various asset volatilities and account-service income rates. Other exogenous parameters are set to the values in Table 1. All numerical values are presented in percentage points.

A. Effects of Asset Volatility

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Asset volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.00</td>
</tr>
<tr>
<td>Charter value</td>
<td>33.64</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>47.55</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>61.91</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>24.18</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>109.46</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>63.40</td>
</tr>
<tr>
<td>Default boundary</td>
<td>63.40</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>2.11</td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.46</td>
</tr>
</tbody>
</table>

B. Effects of Account Service Income

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Account service income rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.30</td>
</tr>
<tr>
<td>Charter value</td>
<td>23.80</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>37.29</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>54.94</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>31.56</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>92.23</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>49.72</td>
</tr>
<tr>
<td>Default boundary</td>
<td>49.72</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>3.07</td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Table 4: Optimal Structure of FDIC-Insured Banks

Numerical values of endogenous variables in the optimal liability structures of FDIC-insured banks are calculated for various asset volatilities and account-service income rates. Other exogenous parameters are set to the values in Table 1. All numerical values are presented in percentage points.

A. Effects of Asset Volatility

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Asset volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.00</td>
</tr>
<tr>
<td>Charter value</td>
<td>35.74</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>61.79</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>49.37</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>24.58</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>111.16</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>63.02</td>
</tr>
<tr>
<td>Default boundary</td>
<td>63.02</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>2.04</td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.45</td>
</tr>
<tr>
<td>Insurance premium</td>
<td>0.06</td>
</tr>
</tbody>
</table>

B. Effects of Account Service Income

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Account service income rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.30</td>
</tr>
<tr>
<td>Charter value</td>
<td>23.80</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>37.29</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>54.94</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>31.56</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>92.23</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>49.72</td>
</tr>
<tr>
<td>Default boundary</td>
<td>49.72</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>3.07</td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.99</td>
</tr>
<tr>
<td>Insurance premium</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Table 5: Optimal Responses to Changes of Bankruptcy Cost

Numerical values of endogenous variables in optimal liability structures are calculated for given exogenous parameters in Table 1, except the bankruptcy cost $\alpha = \beta$ being raised from 25% to 30%. For both an unregulated bank and an FDIC-insured bank, this table reports the optimal values (in percentage points), and their changes, of endogenous variables after raising the bankruptcy cost.

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Before Unregulated</th>
<th>After Unregulated</th>
<th>Change</th>
<th>Before FDIC-insured</th>
<th>After FDIC-insured</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charter value</td>
<td>25.67</td>
<td>24.75</td>
<td>-0.92</td>
<td>27.10</td>
<td>26.48</td>
<td>-0.62</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>37.61</td>
<td>34.22</td>
<td>-3.39</td>
<td>48.43</td>
<td>47.35</td>
<td>-1.08</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>56.97</td>
<td>58.02</td>
<td>1.04</td>
<td>46.74</td>
<td>45.94</td>
<td>-0.80</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>31.09</td>
<td>32.52</td>
<td>1.43</td>
<td>31.93</td>
<td>33.19</td>
<td>1.26</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>94.58</td>
<td>92.24</td>
<td>-2.35</td>
<td>95.17</td>
<td>93.29</td>
<td>-1.88</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>50.15</td>
<td>48.88</td>
<td>-1.26</td>
<td>49.40</td>
<td>48.30</td>
<td>-1.10</td>
</tr>
<tr>
<td>Default boundary</td>
<td>50.15</td>
<td>48.88</td>
<td>-1.26</td>
<td>49.40</td>
<td>48.30</td>
<td>-1.10</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>3.15</td>
<td>3.50</td>
<td>0.35</td>
<td>3.01</td>
<td>3.38</td>
<td>0.37</td>
</tr>
<tr>
<td>Credit spread</td>
<td>1.01</td>
<td>0.94</td>
<td>-0.07</td>
<td>0.97</td>
<td>0.91</td>
<td>-0.06</td>
</tr>
<tr>
<td>Insurance premium</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.11</td>
<td>0.12</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 6: Optimal Responses to Changes of FDIC Operations

Numerical values of endogenous variables in optimal liability structures are calculated for given exogenous parameters in Table 1, except that the FDIC is assumed to offer either efficiency in liquidation (lowering $\beta$ from 25% to 20% while keeping $\alpha = 25\%$) or subsidy in insurance (lowering $\omega$ from 1 to 0.80). For a bank under FDIC, the table reports the optimal values (in percentage points) of endogenous variables before and after lowering $\beta$ or $\omega$. The table also reports the change of each endogenous variable in the optimal responses.

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Baseline</th>
<th>Lower $\beta$</th>
<th>Lower $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>value</td>
<td>change</td>
</tr>
<tr>
<td>Charter value</td>
<td>27.10</td>
<td>27.76</td>
<td>0.67</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>48.43</td>
<td>49.64</td>
<td>1.21</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>46.74</td>
<td>47.58</td>
<td>0.84</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>31.93</td>
<td>30.55</td>
<td>-1.38</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>95.17</td>
<td>97.22</td>
<td>2.05</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>49.40</td>
<td>50.63</td>
<td>1.23</td>
</tr>
<tr>
<td>Default boundary</td>
<td>49.40</td>
<td>50.63</td>
<td>1.23</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>3.01</td>
<td>3.24</td>
<td>0.23</td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.97</td>
<td>1.03</td>
<td>0.07</td>
</tr>
<tr>
<td>Insurance premium</td>
<td>0.11</td>
<td>0.09</td>
<td>-0.02</td>
</tr>
</tbody>
</table>
Table 7: Optimal Responses to Changes of Corporate Tax Rate

Numerical values of endogenous variables in optimal liability structures are calculated for given exogenous parameters in Table 1, except the corporate tax rate $\tau$ being lowered from 30\% to 25\%. For both an unregulated bank and an FDIC-insured bank, this table reports the optimal values (in percentage points), and their changes, of endogenous variables after lowering the corporate tax rate.

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Unregulated before</th>
<th>Unregulated after</th>
<th>Unregulated change</th>
<th>FDIC-insured before</th>
<th>FDIC-insured after</th>
<th>FDIC-insured change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charter value</td>
<td>25.67</td>
<td>20.33</td>
<td>-5.34</td>
<td>27.10</td>
<td>21.74</td>
<td>-5.35</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>37.61</td>
<td>36.60</td>
<td>-1.01</td>
<td>48.43</td>
<td>47.12</td>
<td>-1.31</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>56.97</td>
<td>51.12</td>
<td>-5.86</td>
<td>46.74</td>
<td>41.17</td>
<td>-5.57</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>31.09</td>
<td>32.62</td>
<td>1.53</td>
<td>31.93</td>
<td>33.45</td>
<td>1.53</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>94.58</td>
<td>87.71</td>
<td>-6.87</td>
<td>95.17</td>
<td>88.29</td>
<td>-6.88</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>50.15</td>
<td>48.80</td>
<td>-1.35</td>
<td>49.40</td>
<td>48.07</td>
<td>-1.33</td>
</tr>
<tr>
<td>Default boundary</td>
<td>50.15</td>
<td>48.80</td>
<td>-1.35</td>
<td>49.40</td>
<td>48.07</td>
<td>-1.33</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>3.15</td>
<td>2.90</td>
<td>-0.25</td>
<td>3.01</td>
<td>2.78</td>
<td>-0.24</td>
</tr>
<tr>
<td>Credit spread</td>
<td>1.01</td>
<td>0.94</td>
<td>-0.07</td>
<td>0.97</td>
<td>0.90</td>
<td>-0.07</td>
</tr>
<tr>
<td>Insurance premium</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.11</td>
<td>0.10</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

Table 8: Optimal Responses to Changes of Capital Requirement

Numerical values of endogenous variables in optimal liability structures are calculated for given exogenous parameters in Table 1, when unregulated banks are subject to no regulatory equity requirement and requirements ranging of 4, 7, 10, 15, and 20 percent of total assets. The optimal values of the endogenous variables are presented in percentage points.

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>Regulatory equity requirement None</th>
<th>4.00</th>
<th>7.00</th>
<th>10.00</th>
<th>15.00</th>
<th>20.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charter value</td>
<td>25.67</td>
<td>25.67</td>
<td>25.64</td>
<td>25.45</td>
<td>24.86</td>
<td>24.01</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>37.61</td>
<td>37.61</td>
<td>36.58</td>
<td>34.76</td>
<td>32.01</td>
<td>29.52</td>
</tr>
<tr>
<td>Sub-debt ratio</td>
<td>56.97</td>
<td>56.97</td>
<td>56.42</td>
<td>55.24</td>
<td>52.98</td>
<td>50.48</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>31.09</td>
<td>31.09</td>
<td>32.64</td>
<td>35.46</td>
<td>39.86</td>
<td>44.02</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>94.58</td>
<td>94.58</td>
<td>93.00</td>
<td>90.00</td>
<td>85.00</td>
<td>80.00</td>
</tr>
<tr>
<td>Closure boundary</td>
<td>50.15</td>
<td>50.15</td>
<td>48.77</td>
<td>46.35</td>
<td>42.68</td>
<td>39.35</td>
</tr>
<tr>
<td>Default boundary</td>
<td>50.15</td>
<td>50.15</td>
<td>48.77</td>
<td>46.35</td>
<td>42.68</td>
<td>39.35</td>
</tr>
<tr>
<td>Bankruptcy loss</td>
<td>3.15</td>
<td>3.15</td>
<td>2.90</td>
<td>2.49</td>
<td>1.94</td>
<td>1.52</td>
</tr>
<tr>
<td>Credit spread</td>
<td>1.01</td>
<td>1.01</td>
<td>0.94</td>
<td>0.82</td>
<td>0.67</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Figure 1: An Illustration of Bank Liability Structure

<table>
<thead>
<tr>
<th>Asset Side</th>
<th>Liability Side</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets:</strong> $V$</td>
<td><strong>Deposits:</strong> $D$</td>
</tr>
<tr>
<td>Volatility: $\sigma$</td>
<td>Benefit: deduct tax $\tau$</td>
</tr>
<tr>
<td>Cash flow: $\delta$</td>
<td>Benefit: account service $\eta$</td>
</tr>
<tr>
<td>Charter value: $F - V$</td>
<td>Cost: bankruptcy $\alpha$ or $\beta$</td>
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<tr>
<td><strong>Subordinated Debt:</strong> $D_1$</td>
<td>Cost: bankruptcy $\alpha$ or $\beta$</td>
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Tangible equity: $V - (D + D_1)$

Bank value: $F = D + D_1 + E$

Equity: $E$
Figure 2: Asset Volatility of Financial Firms

A. Asset Volatilities of Banks

B. Asset Volatilities of Manufacturing Firms
Figure 3: Bankruptcy of Unregulated Banks

- Bank-run in fixed structure
- Default in fixed structure
- Bank-run in optimal structure
- Default in optimal structure

Bankruptcy Cost vs Bankruptcy Boundary
Figure 4: Bankruptcy under FDIC

![Graph showing bankruptcy boundaries with different structures: closure in fixed structure, default in fixed structure, closure in optimal structure, default in optimal structure.](image-url)

Legend:
- Dashed line: Closure in fixed structure
- Circle: Default in fixed structure
- Solid line: Closure in optimal structure
- Asterisk: Default in optimal structure
Figure 5: Insurance Premium, Closure Rule, and Volatility
Figure 6: Incentives of Risk Shifting

![Diagram showing incentives of risk shifting with equity value as a function of asset volatility. The graph compares different scenarios with constant and varying premiums and Omega values.]